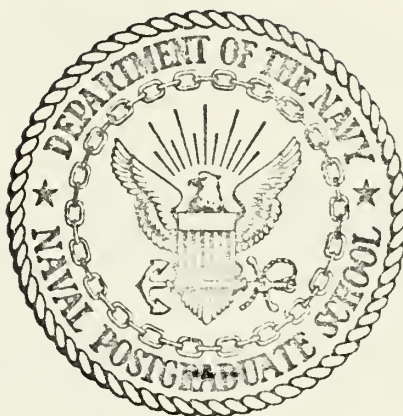


EVALUATION OF MINIMUM AIRCRAFT FLYING
SPEED BY DIGITAL SIMULATION

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THESIS

Evaluation of Minimum Aircraft Flying
Speed by Digital Simulation

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Speed by Digital Simulation

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ABSTRACT

Aircraft minimum flying speed, as determined by actual flight test, is published in aircraft handbooks for pilot guidance. The test flight results are used to determine and confirm take-off and landing speeds, field lengths, left-hand portion of the maneuvering envelopes (V-n diagram), etc.. Determination of the absolute minimum flying speed of an aircraft on the other hand, has not been of prime importance in flight test.

In the present analysis digital simulation allowed the systematic study of not only the minimum flying speed as defined by Federal Aviation Regulations but also the absolute minimum flying speed attainable in steady, unaccelerated flight. The study included such effects as deceleration rate, rate of change of elevator angle, aircraft weight and pitch moment of inertia.

It was found for an assumed light-weight fighter aircraft that the absolute minimum flying speed was approximately 20 knots less than the FAR minimum flying speed. Moreover the FAR minimum flying speeds tended to be quite sensitive to rate of change of elevator angle.



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SYMBOLS

$a_{n_{fp}}$	normal flight path acceleration, ft/sec ²
B	pitch moment of inertia, slug-ft ²
C	location of the aircraft center of mass
\bar{c}	mean aerodynamic chord
C_D	drag coefficient, drag/ $\bar{q}S$
C_L	lift coefficient, Lift/ $\bar{q}S$
$C_{L_{max}}$	maximum lift coefficient
C'_L	apparent lift coefficient, weight/ $\bar{q}S$
$C'_{L_{max}}$	maximum apparent lift coefficient
$C_L(\alpha)$	static lift coefficient as a function of angle of attack
$C_{L_{\delta_e}}$	rate of change of lift coefficient for a change in elevator angle
$C_{L_{\dot{\theta}}}$	rate of change of lift coefficient for a change in pitch velocity.
C_M	pitching moment coefficient, moment/ $\bar{q}S\bar{c}$
$C_M(\alpha)$	static pitching moment coefficient as a function of angle of attack
$C_{M_{\dot{\alpha}}}$	angle of attack damping derivative, $\frac{\partial C_M}{\partial \left(\frac{\dot{\alpha} \bar{c}}{2V} \right)} (\text{rad}^{-1})$
$C_{M_{\delta_e}}$	pitching moment coefficient derivative for elevator deflection
$C_{M_{\dot{q}}}$	pitch damping derivative, $\frac{\partial C_M}{\partial \left(\frac{\dot{\theta} \bar{c}}{2V} \right)} (\text{rad}^{-1})$
C_T	thrust coefficient, thrust/ $\bar{q}S$
C_X	longitudinal aerodynamic force coefficient, body axis
C_Z	vertical aerodynamic force coefficient, body axis

D	product of inertia $\int yz dm$, slug-ft ²
δ_e	elevator angle, deg.
$\dot{\delta}_e$	rate of change of elevator angle, deg/sec.
e_t	local truncation error for Runge-Kutta algorithm
F	product of inertia $\int xy dm$, slug-ft ²
\bar{F}	resultant external force vector relative to aircraft center of mass
\bar{G}	resultant external moment vector about the aircraft center of mass
g	scalar acceleration due to gravity, ft/sec ²
\bar{h}	angular momentum vector relative to aircraft center of mass
$[L, M, N]$	scalar components of \bar{G} ; rolling, pitching and yawing moments, ft-lb.
$[P, Q, R]$	scalar components of \bar{W} ; rolling, pitching and yawing velocities, rad/sec.
\dot{Q}	rate of change of pitching velocity
\bar{q}	dynamic pressure, lb/ft ²
\bar{q}_s	dynamic pressure at minimum flying speed/stall speed, lb/ft ²
S	wing area, ft ²
T	thrust, lb.
$[u, v, w]$	scalar components of \bar{V}_C , relative to body axis system, ft/sec.
\dot{U}	rate of change of V , ft/sec ²
\dot{V}	rate of change of V_C , as determined by FAR method
\dot{V}_n	rate of change of V_C , as determined by altitude break method
V_C	magnitude of resultant linear velocity of aircraft center mass, ft/sec.
\bar{V}_C	resultant velocity vector of aircraft center of mass, ft/sec

$V_{c_{min}}$	minimum flying speed as determined by C_L method, ft/sec.
$V_{c'_{min}}$	minimum flying speed as determined by C_L' method, ft/sec.
$V_{h_{min}}$	minimum flying speed as determined by altitude break method, ft/sec..
V_{min}	minimum flying speed as determined by constant deceleration rate, ft/sec..
W	aircraft weight, lb..
\dot{W}	rate of change of W , scalar component of \overline{V}_c , ft/sec ² .
$[x,y,z]$	cartesian coordinate axes notation for body axis system, origin located at aircraft center of mass.
$[x',y',z']$	cartesian coordinate axes notation for stability axes system, origin located at aircraft center of mass.
$[X,Y,Z]$	components of resultant aerodynamic forces acting on aircraft, lb..
$Y_{n+1,1}$	variable in Runge-Kutta algorithm obtained by integrating between two points, X_n and X_{n+1} , with step size h_1 .
$Y_{n+1,2}$	variable in Runge-Kutta algorithm obtained by integrating between two points, X_n and X_{n+1} , with step size h_2 where $h_2 = h_1/2$.
α	angle of attack, deg..
$\dot{\alpha}$	rate of change of angle of attack, deg/sec..
θ	aircraft pitch angle, angle between horizontal reference and the X axis of the body axis system, deg.
$\dot{\theta}$	rate of change of pitch angle, deg/sec.
ρ	air density, slugs/ft ³
$[\psi,\theta,\phi]$	Euler angles, rad.
$\overline{\omega}$	angular velocity vector of the aircraft, rad/sec.

I.. INTRODUCTION

Minimum flying speed and or stalling speed is defined by Federal Aviation Regulations (FAR) [Ref. 1] as the minimum steady speed at which the aircraft remains controllable. The Naval Test Pilot School Flight Test Manual [Ref. 2] defines minimum flying speed as the minimum steady airspeed attainable in unaccelerated flight or the minimum usable airspeed. The actual determination of minimum flying speed is always accomplished through flight tests. Methods for theoretical predictions are empirical, based upon wind-tunnel results, experience, etc.

FAR specified that stalls would be demonstrated by trimming the aircraft at 130 percent of the estimated minimum flying speed and decelerating at a constant rate until a minimum speed was obtained. The certified stall speed would correspond to a deceleration rate of 1 knot/sec. To facilitate the test pilot's task, Ref. 1 and 2 defined aircraft characteristics and parameters indicative of minimum flying speed. Typical indications were: longitudinal, directional or lateral divergence, excessive altitude loss, loss of control/effectiveness, etc. Data analysis methods and restrictions on demonstration technique, other than rate of change of airspeed, were not specified by Ref. 1.

Additional flight test techniques have been proposed and evaluated by the National Aeronautics and Space Administration [Refs. 3 and 4]. These studies, through flight tests, evaluated three demonstration techniques: FAR, 1-g break and constant-rate-of-climb. Three methods of data-analysis were used with the FAR demonstration technique to

determine minimum flying speed:: constant deceleration, C_L^1 and C_L^2 methods. The minimum flying speeds obtained by the three techniques were corrected to a deceleration rate of 1 knot/sec in accordance with Ref. 11.

The study described herein applied numerical methods to simulate flight test data in an attempt to define an absolute minimum flying speed. Five data-analysis methods were used. The speed was an absolute minimum obtainable for steady, unaccelerated flight when the FAR correction to a 1 knot/sec. deceleration rate was disallowed. The absolute minimum flying speeds were compared with FAR certified minimum flying speeds, i.e., corrected to 1 knot/sec. deceleration rates.

"Flight test data" for the F-94A, a small single-engine jet fighter, was obtained from a computer program of the non-linear equations of aircraft motion. FORTRAN IV language and the Naval Postgraduate School IBM 360 digital computer were utilized. The computer program provided numerous options for trimming and flying the aircraft, including variations of thrust, weight, rate of change of elevator angle and flight orientation, i.e., climbing, level or descending flight.

For this study, the aircraft was trimmed in steady, descending flight at 130 percent of the estimated minimum flying speed and decelerated by decreasing the elevator angle at a constant rate. Two values for thrust were used to trim and stall the aircraft: $T/W \leq 0.02$ and $T/W = 0.11$.

The effects on minimum flying speed of thrust, aircraft weight, rate of change of elevator angle and aircraft moment of inertia about the lateral axis were also studied.

The purpose of this study was to investigate the relationships of the aircraft minimum flying speeds as determined by the different data-analysis methods.

II.. EXPERIMENTAL METHODS

A. BASIC APPROACH

The study of minimum flying speed was restricted to aircraft motion in the XZ plane: i.e., to aircraft motion along the X and Z axes and pitching moments about the Y axis. It was assumed that no adverse lateral or directional traits occurred while decelerating to the minimum flying speed.

Non-linear equations of aircraft motion with three degrees-of-freedom were derived from Euler's equations of motion,, Appendix A. Extensive use was made of Etkin [Ref. 5]. The equations were set up for input of non-linear aerodynamic data,, by means of a table look-up procedure.

A computer program was written in FORTRAN IV language for the Naval Postgraduate School IBM 360 digital computer,, Appendix B. The program employed a fourth-order Runge-Kutta algorithm in solving the equations of motion. Real-time was used in evaluating the aircraft motion from approximately $1.3 V_{min}$ to V_{min} . Typical time histories of the aircraft parameters were plotted in Figure 1. Figure 2 contains summary plots of V_{min} as a function of deceleration rate for different rates of change of elevator angle. The thrust to weight ratio (T/W) was 0.02.

B. TEST AIRCRAFT

The F-94A, a small single-engine jet fighter,, was utilized as the test aircraft.

Reference aerodynamic data were obtained from Blakelock [Ref. 5] in the form of linear stability derivatives for all terms except those

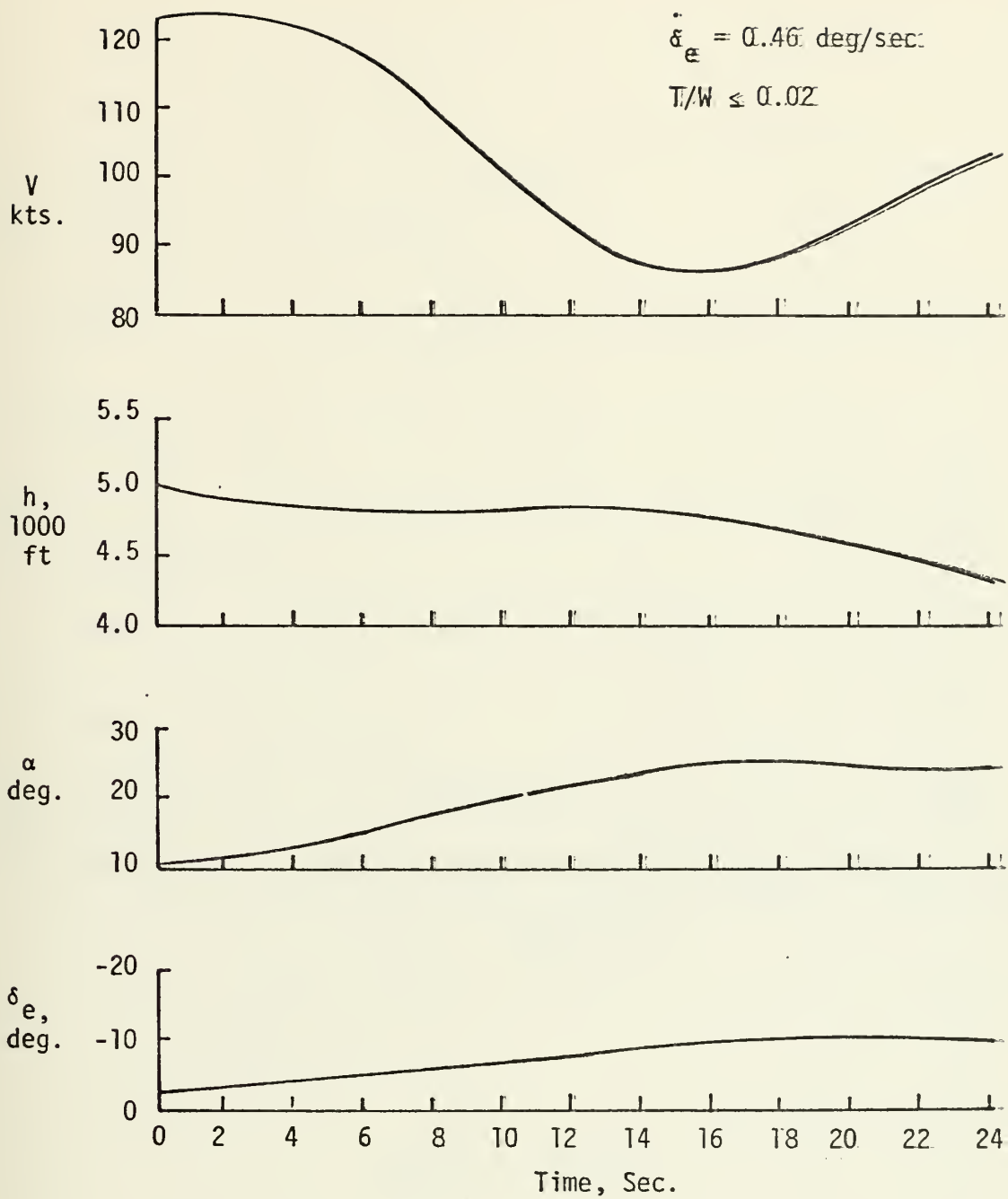


Figure 1. Typical Time Histories of Aircraft Parameters.

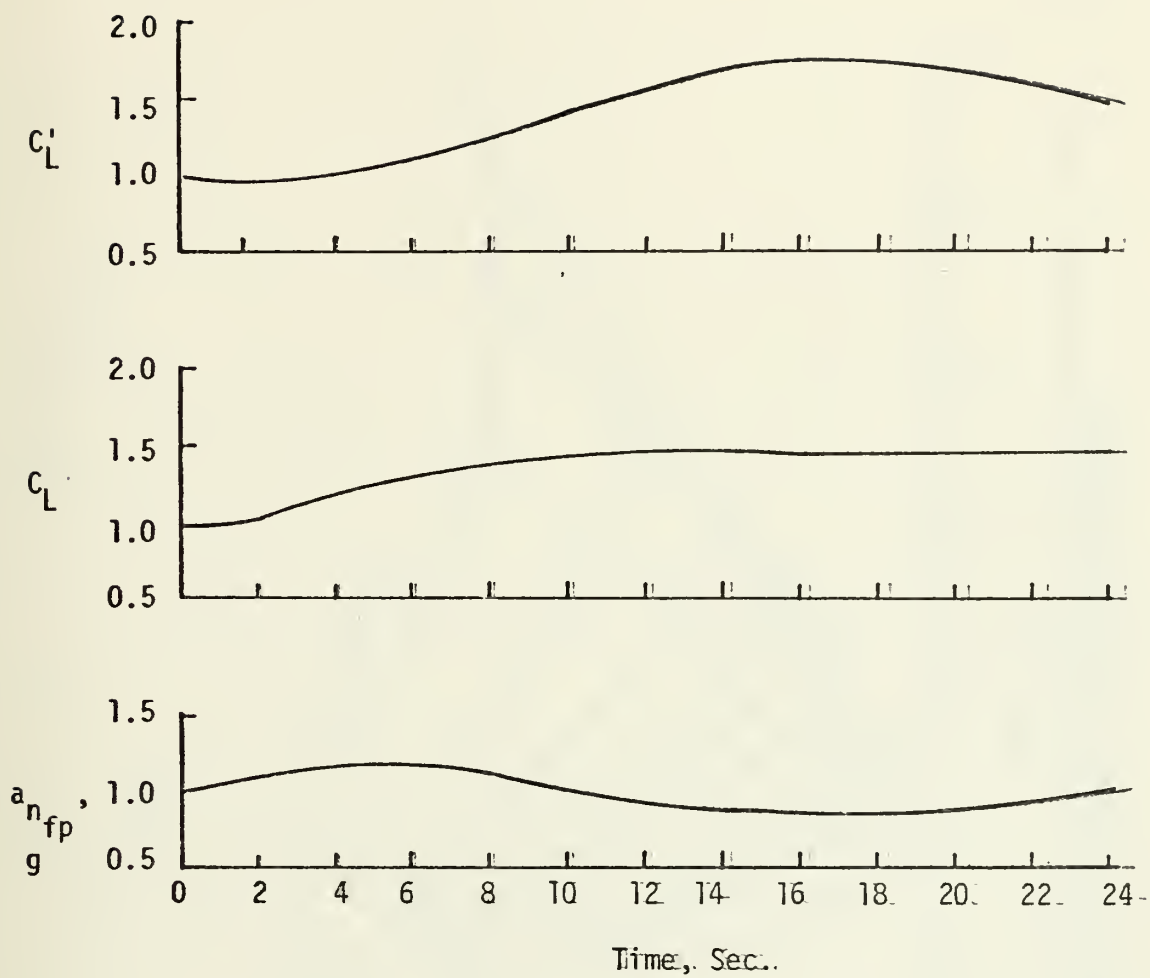


Figure 1. (Cont'd)

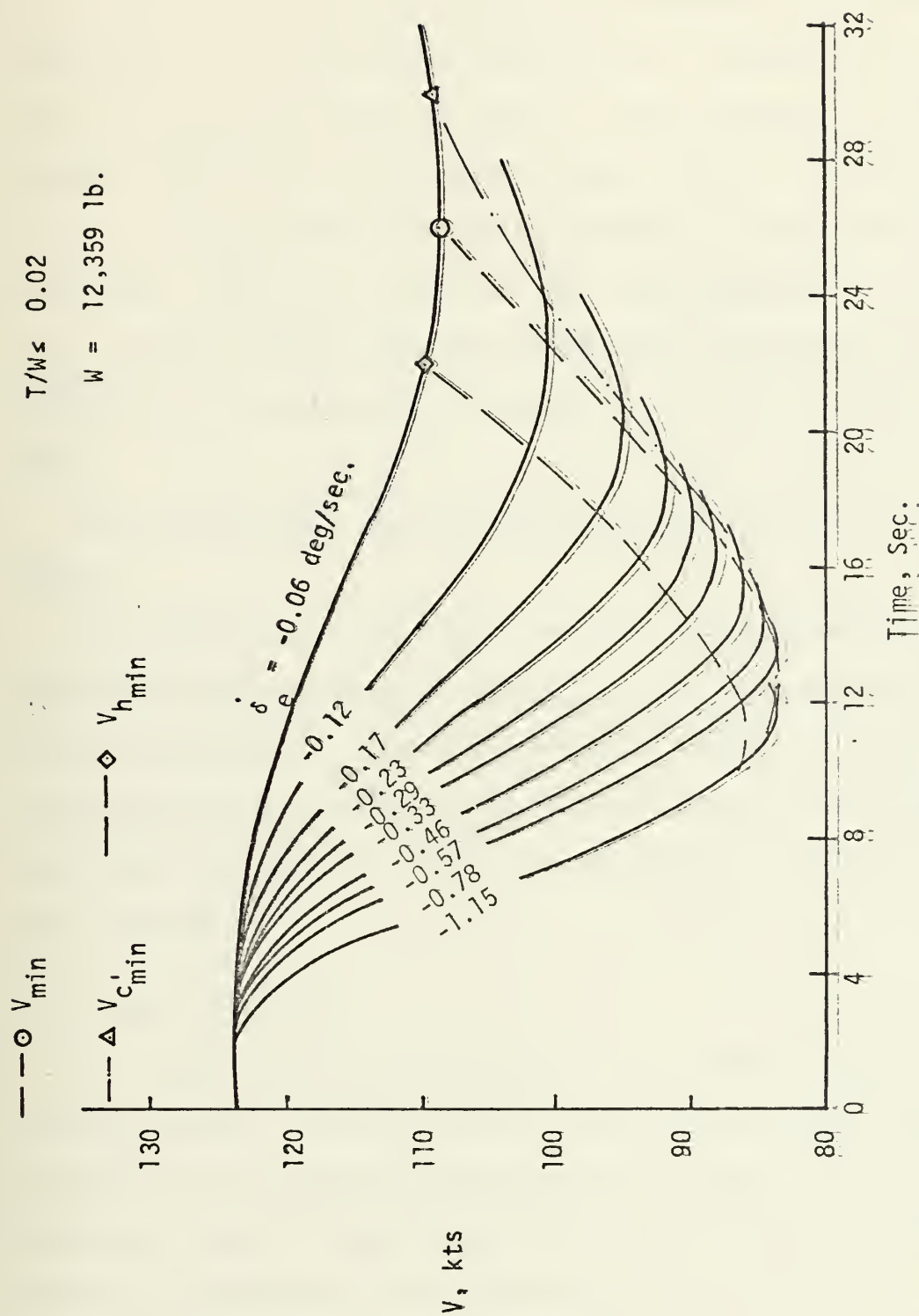


Figure 2. Time History Trends of Aircraft Deceleration, Elevator Varried From -0.06 to -1.15 deg/sec. To Obtain Plots, δ_e Constant During Each Maneuver.

depending upon angle of attack. The variation of the complete aircraft C_L , C_D and C_M was estimated from the available data for a center of gravity located at the 0.252 \bar{C} (wing mean aerodynamic chord) station and an airplane in the landing configuration. Although the variations of C_L , C_D and C_M with angle of attack, α , were estimated for zero elevator deflection, the use of the linear stability derivatives such as $C_{L_{\delta_e}}$ and $C_{M_{\delta_e}}$ allowed trimming the aircraft to any initial flight conditions. The C_L vs. α curve for $\delta_e = 0$ was selected for a trim $C_{L_{\max}}$ of about 1.46 with the stall break being very gradual, which is indicative of a progressive flow separation starting at the wing trailing edge.

Aircraft reference data and physical characteristics are listed in Table I.

The choice of aircraft flight traits are arbitrary and may seem restrictive to a person who is solely concerned about a particular aircraft by virtue of personal experiences. However, it should be recognized that the methods and qualities described in this analysis are typical and may be modified to a particular application by changing the input data.

C. FLIGHT PROCEDURES

All "test flights" were conducted with the F-94A configured for a landing approach. The aircraft was trimmed for steady descending flight at 130 percent of the estimated minimum flying speed. Aircraft thrust, weight and moment of inertia about the lateral axis were the only physical characteristics changed during the "test flights".

TABLE I

a. Physical Data

<u>Parameter</u>	<u>Value</u>
S	239.9 ft^2
\bar{c}	664.4 ft
W	$12,359 \text{ lb}$
I_{yy}	$26,545 \text{ slug-ft}^2$
$C_{L_{\alpha}}$ (Linear region)	5.27 rad^{-1}
$C_{L_{\delta e}}$	0.433 rad^{-1}
$C_{M_{\alpha}}$ (Linear region)	-0.406 rad^{-1}
$C_{M_{\delta e}}$	-0.88 rad^{-1}
$C_{M_{\dot{\alpha}}}$	-4.268 rad^{-1}
C_{M_q}	-8.165 rad^{-1}
C	$25.2\% \bar{c}$

TABLE I (Cont'd)

b. Tabular Data (complete aircraft, $\delta_e = 0$)

α (rad)	C_L	C_M	C_{D_e}
0.10472	0.552	-0.0027	0.0940
0.13963	0.738	-0.0170	0.1150
0.17453	0.921	-0.0315	0.1400
0.20944	1.093	-0.0445	0.1695
0.24435	1.233	-0.0585	0.2030
0.27925	1.346	-0.0740	0.2490
0.31416	1.430	-0.0910	0.3120
0.34907	1.484	-0.1120	0.3640
0.38397	1.513	-0.1320	0.4050
0.41888	1.520	-0.1520	0.4550
0.45379	1.500	-0.1720	0.4930
0.48869	1.462	-0.2075	0.5350
0.52360	1.415	-0.2400	0.5720
0.55851	1.360	-0.2795	0.6100

Deceleration from the trim condition was accomplished by decreasing the elevator angle, to simulate aft control stick movement. Available time histories, [Ref. 4], indicated elevator angle to be a near-linear function of real-time during the deceleration maneuver. Therefore, a constant rate of change of elevator angle was chosen for this study. Rate of change of elevator angle was varied between -0.05 and -1.15 deg/sec. for the different maneuvers.

The effects of thrust were studied by making two "test flights" for each elevator angle schedule. Two thrust values were used for the "flights" which gave thrust to weight ratios of 0.02 and 0.11 .

Five "flights" were made at -0.286 deg/sec. rates of change of elevator angle while varying aircraft weight to study the effects of aircraft weight on minimum flying speed. The aircraft weight was varied from $12,359$ pounds to $16,359$ pounds in increments of 1000 pounds.

"Flights" were made at representative rates of change of elevator angle to study the effects of aircraft moment of inertia about the lateral axis, on minimum flying speed. Values 25 percent above and below the true aircraft moment of inertia were evaluated.

Data from the original "test flight" indicated the possible existence of an optimum elevator schedule in determining minimum flying speed. An exponential elevator schedule was studied and compared to the linear elevator schedule. The aircraft was decelerated using a linear schedule until the deceleration rate fell below a preselected rate. At this point the elevator schedule was changed to an exponential schedule until minimum flying speed was obtained.

D. DATA-ANALYSIS METHODS

Five data-analysis methods were used to determine minimum flying speed from the two main sets of "flight test" data. These methods were:

1. Constant Deceleration Rate

The linear elevator angle schedule resulted in near constant deceleration rates by the test aircraft. Minimum flying speed was defined as the actual minimum speed obtained during the deceleration. For consistency, a deceleration rate was defined as the slope of a straight line drawn from V_{min} to T.L. V_{min} . This deceleration rate was used in both the C_L and C'_L methods. Plots were made of minimum flying speed as a function of deceleration rate and minimum flying speed as a function of rate of change of elevator angle for the two values of thrust.

The constant deceleration rate method was used to analyze data when the effects of aircraft weight and moment of inertia on minimum flying speed were studied.

2. C'_L Methods

C'_L was defined as the aircraft lift coefficient independent of normal flight path acceleration:

$$C'_L = \frac{W}{\bar{q}S}$$

The assumption of aircraft weight and aircraft lift being equivalent throughout the maneuver corresponds to considering C'_L as being an apparent lift coefficient rather than a true value. Minimum flying speed was defined by the C'_L method as the speed of the aircraft when C'_L was at a maximum value. Plots were made of C'_L as a function of $C'_{L_{max}}$

deceleration rate, $V_{C_{\max}}$ as a function of deceleration rate and $V_{C_{\min}}$ as a function of rate of change of elevator angle.

3. C_L Method

C_L was defined as being the true aircraft lift coefficient which compensated for effects of normal flight path acceleration:

$$C_L = \frac{W \cdot a_{h_{fp}}}{\bar{q} S}$$

Minimum flying speed was defined by the C_L method as the speed of the aircraft when C_L was at a maximum value. Plots were made of $C_{L_{\max}}$ as a function of deceleration rate, $V_{C_{\min}}$ as a function of deceleration rate and $V_{C_{\min}}$ as a function of rate of change of elevator angle.

4. Altitude Break Method

The aircraft altitude trace had a relatively linear characteristics slope during the deceleration maneuver. With the onset of stall the characteristic slope would increase significantly in value. The minimum flying speed was defined as the speed of the aircraft where the linear characteristic slope of the altitude trace could no longer be maintained. The characteristic slope increase was indicative of increased altitude loss.

A deceleration rate was defined as the slope of a straight line drawn from $V_{h_{\min}}$ to $1.1 V_{h_{\min}}$. Plots were made of $V_{h_{\min}}$ as a function of rate of change of elevator angle.

5. 1-g Break Method

The minimum flying speed defined by this method was the last speed at which the aircraft could maintain 1-g flight. This method proved unreliable in data reduction for an aircraft trimmed in steady,

descending flight and was thus not pursued further.. For this method,,
an aircraft must be decelerated from an initial climb condition:.

III.. RESULTS AND DISCUSSION

A. ABSOLUTE MINIMUM FLYING SPEED

1. Constant Deceleration Rate Method

The minimum flying speeds determined were plotted in Figure 3 as a function of deceleration for thrust-to-weight ratios, 0.02 and 0.11. For a thrust-to-weight ratio of 0.02 the absolute minimum flying speed was 83.7 knots and occurred at a deceleration rate of 2160 knots/sec. A thrust-to-weight ratio of 0.11 decreased the absolute minimum flying speed to 80.6 knots with no appreciable change in deceleration rate. Figure 2 also indicated absolute minimum flying speed was sensitive to variation in optimum deceleration rate. The addition of thrust reduced this sensitivity somewhat.

Figure 4 indicated absolute minimum flying speed was relatively insensitive to rate of change of elevator. Rates between -0.7 and -1.1 deg/sec. were able to give a close approximation of absolute minimum flying speed.

2. C_L' Method

Maximum values of C_L' were plotted in Figure 5 as a function of deceleration rate. $V_{C_{L'}'_{min}}$ was plotted as a function of deceleration rate in Figure 6 and as a function of rate of change of elevator angle in Figure 7. A C_L' of 1.815 was the maximum attainable in steady, unaccelerated flight. The absolute minimum flying speed was 83.5 knots. The thrust-to-weight ratio of 0.11 decreased the absolute minimum flying speed three knots with minimal effects on deceleration rate. $V_{C_{L'}'_{min}}$ was fairly sensitive to deceleration rate and insensitive to rate of change of elevator angle.

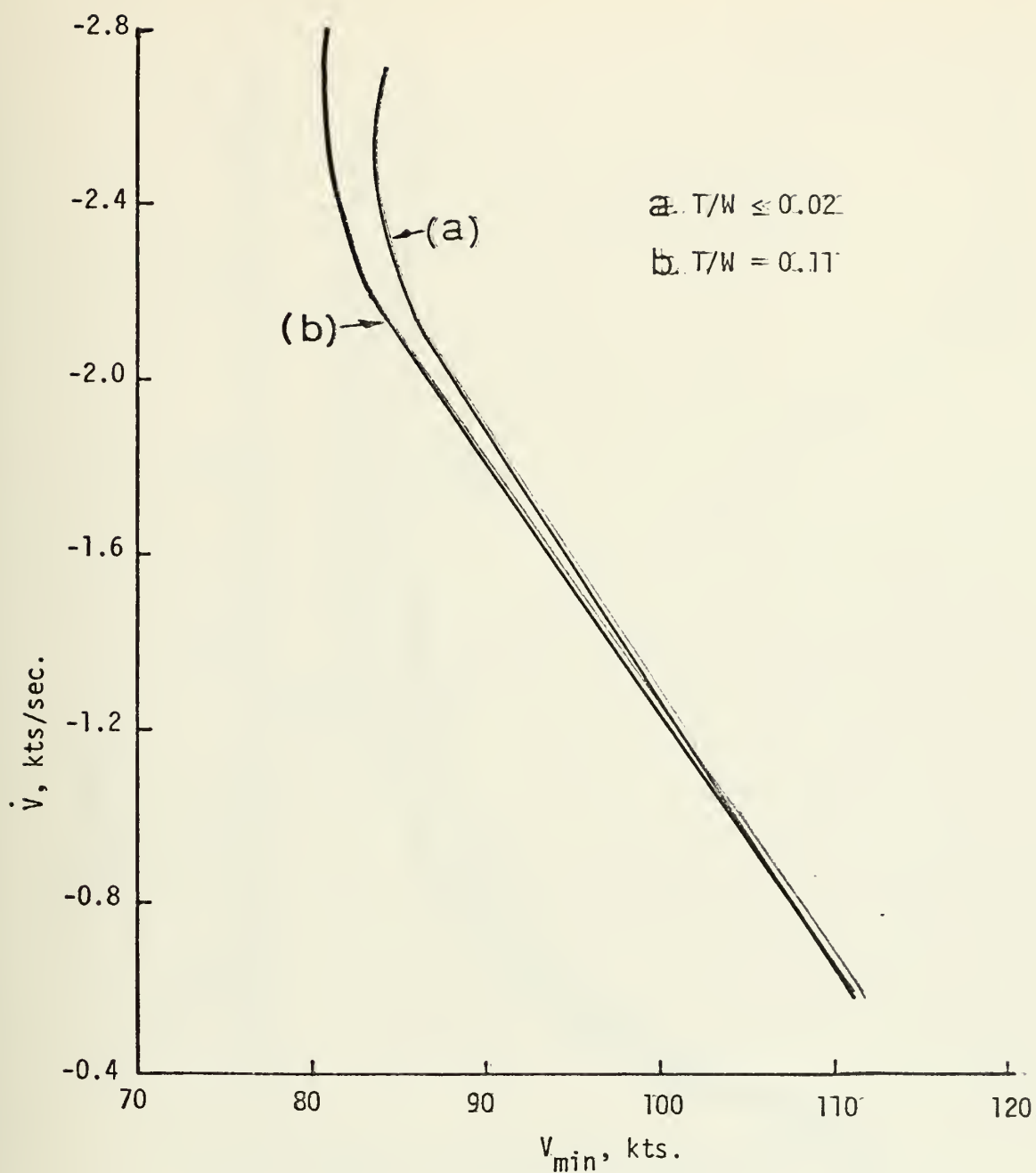


Figure 3. Minimum Flying Speed as a Function of Deceleration Rate.

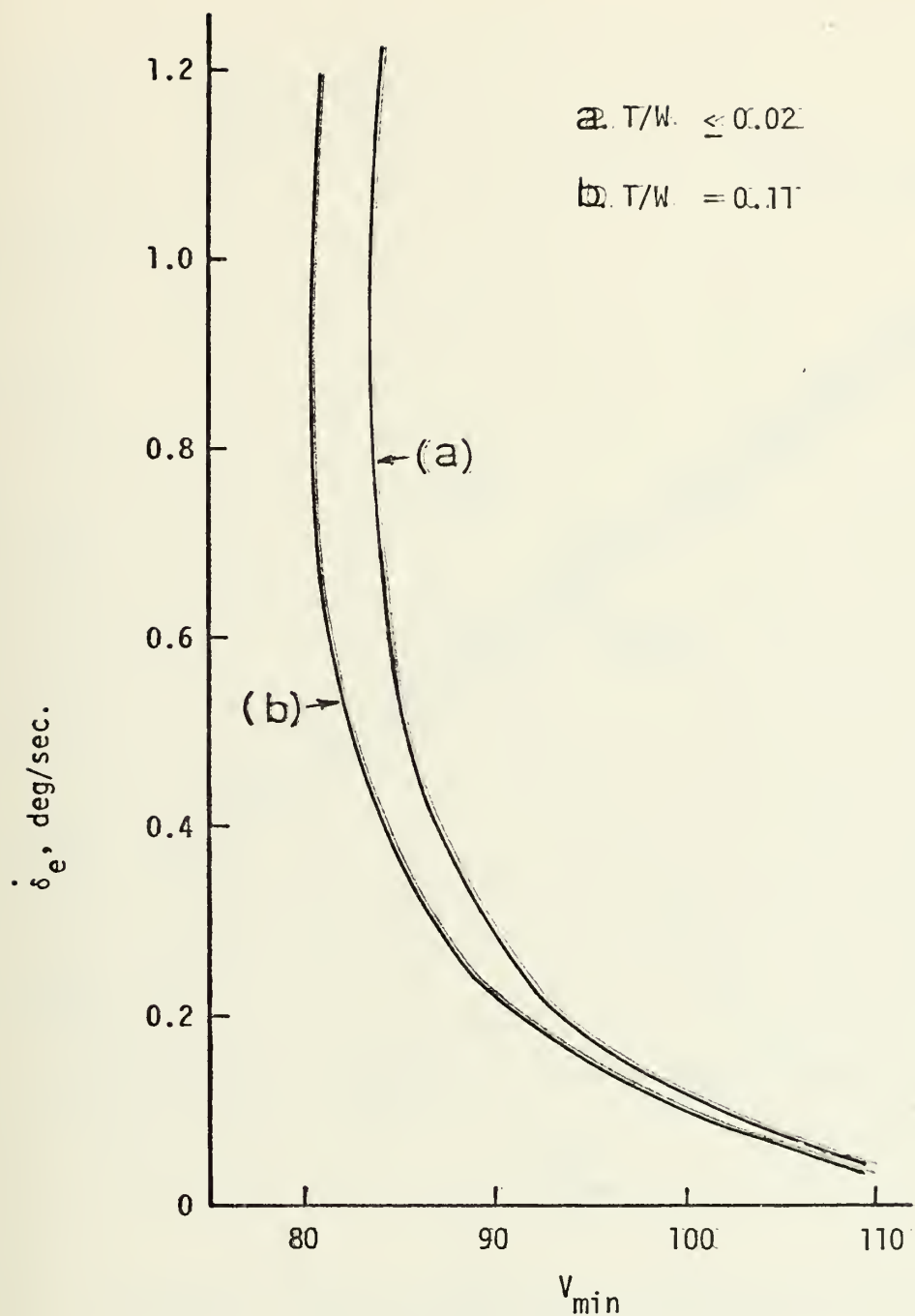


Figure 4. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

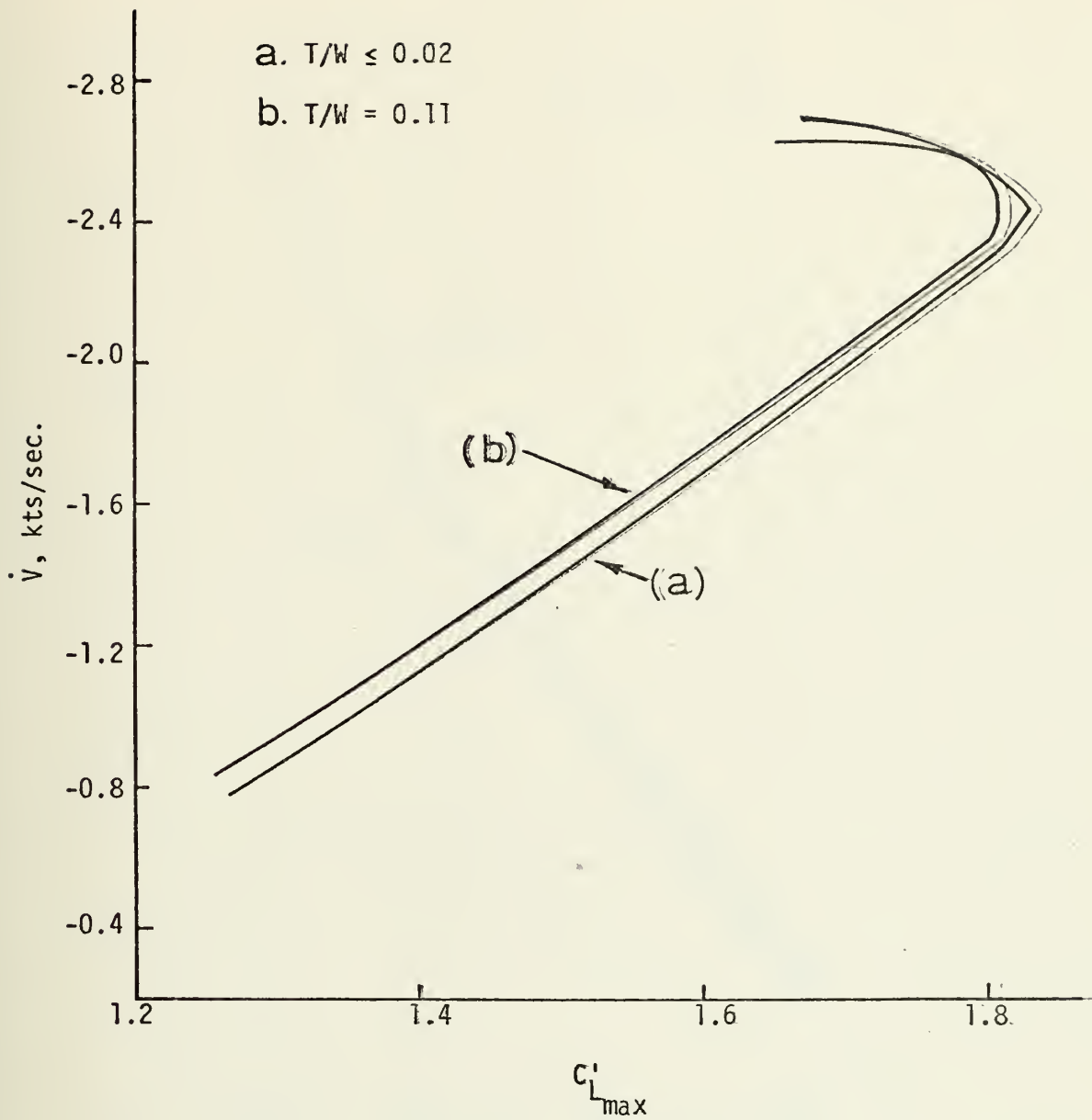


Figure 5. Apparent Maximum Lift Coefficient as a Function of Deceleration Rate.

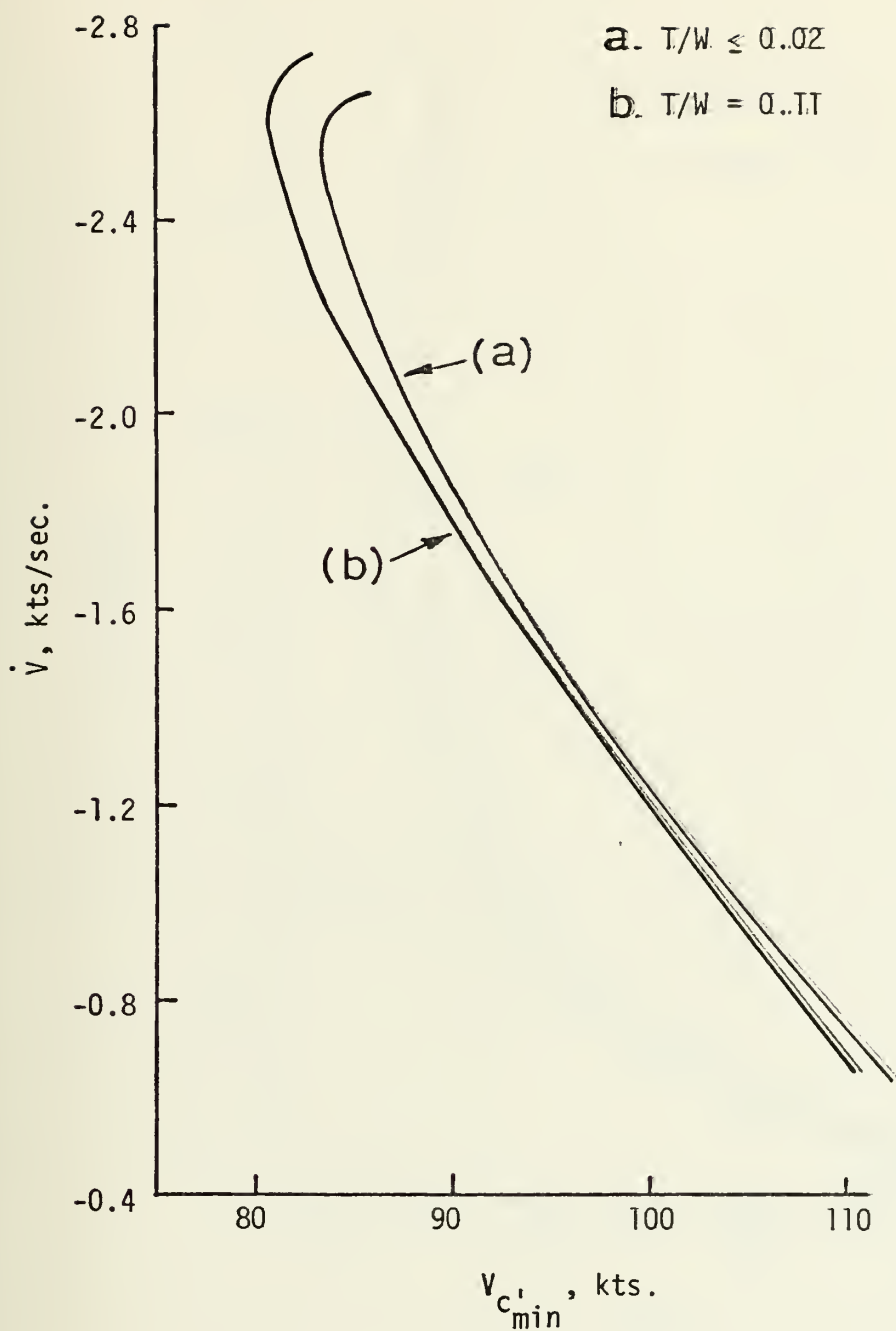


Figure 6. Minimum Flying Speed as a Function of Deceleration Rate.

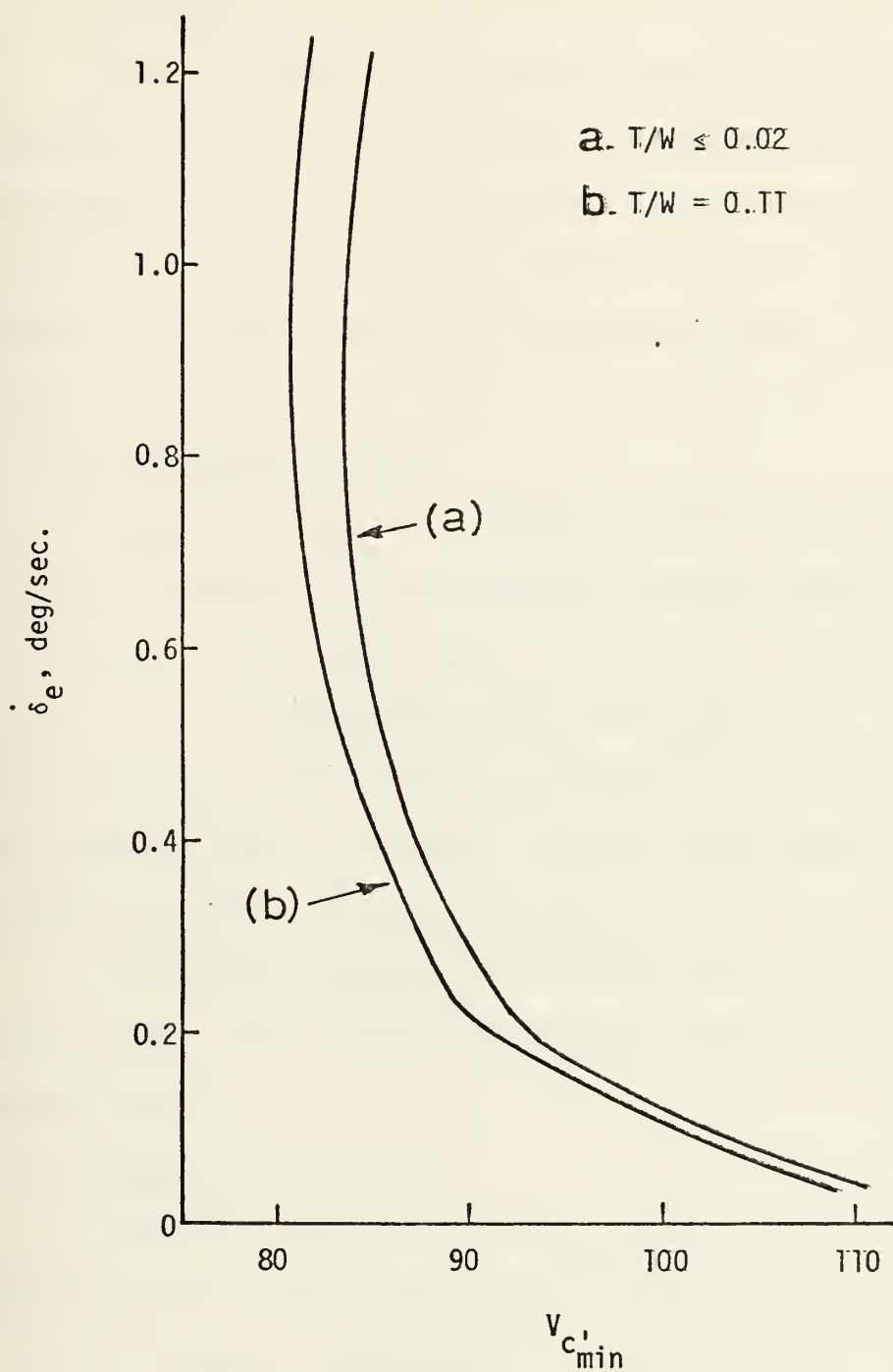


Figure 7. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

3. C_L Method

Absolute minimum flying speed was 88 knots for a thrust-to-weight ratio of 0.02, Figures 8 and 9. Increasing thrust decreased this speed 2.5 knots. $V_{C_{min}}$ was sensitive to both deceleration rate and rate of change of elevator angle. Figure 10 indicated the absolute maximum C_L value was insensitive to deceleration rate and increased thrust.

4. Altitude Break Method

$V_{h_{min}}$ was plotted as a function of deceleration rate in Figure 11 and as a function of rate of change of elevator angle in Figure 12. The absolute minimum flying speed for a thrust-to-weight ratio of 0.02 was 83.5 knots at a deceleration rate of -2.6 knots/sec.. Figure 11 indicated that two values for $V_{h_{min}}$ existed for one value of deceleration rate between -2.6 and -3.76 kts/sec. Figure 12 indicated $V_{h_{min}}$ was insensitive to rate of change of elevator angle.

The altitude break method was dependent on graphical interpretation for data. This accounts for the small amount of data scatter in Figures 11 and 12.

5. Comparison of Methods

The general relationships of absolute minimum flying speed as determined by the various methods are indicated in Figure 13.

The constant deceleration, C_L and altitude break methods defined the absolute minimum flying speed to be 83 knots, Figure 13. The C_L method absolute minimum flying speed was 88 knots. Increased thrust decreased the absolute minimum flying speed defined by the four methods about three knots.

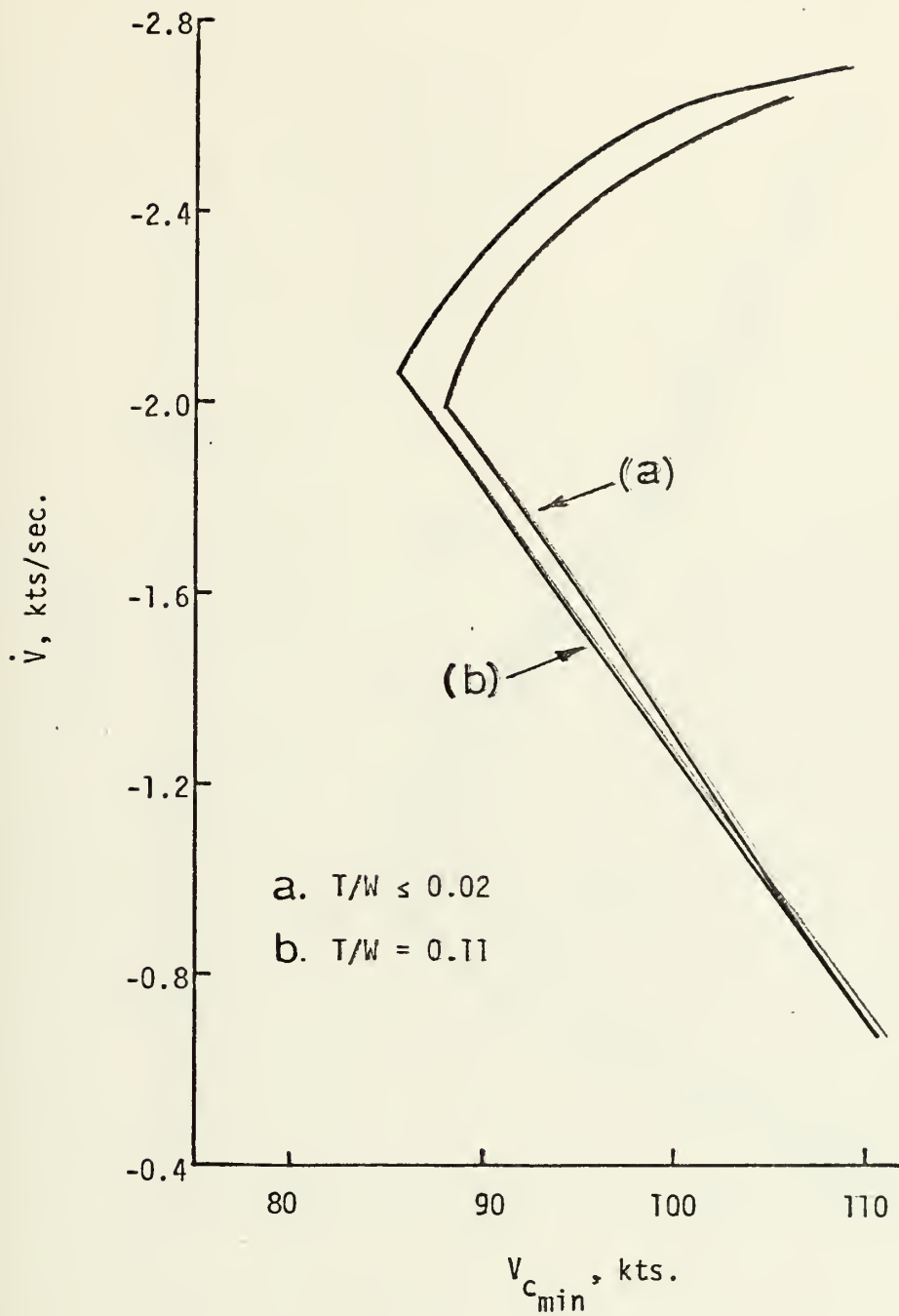


Figure 8. Minimum Flying Speed as a Function of Deceleration Rate.

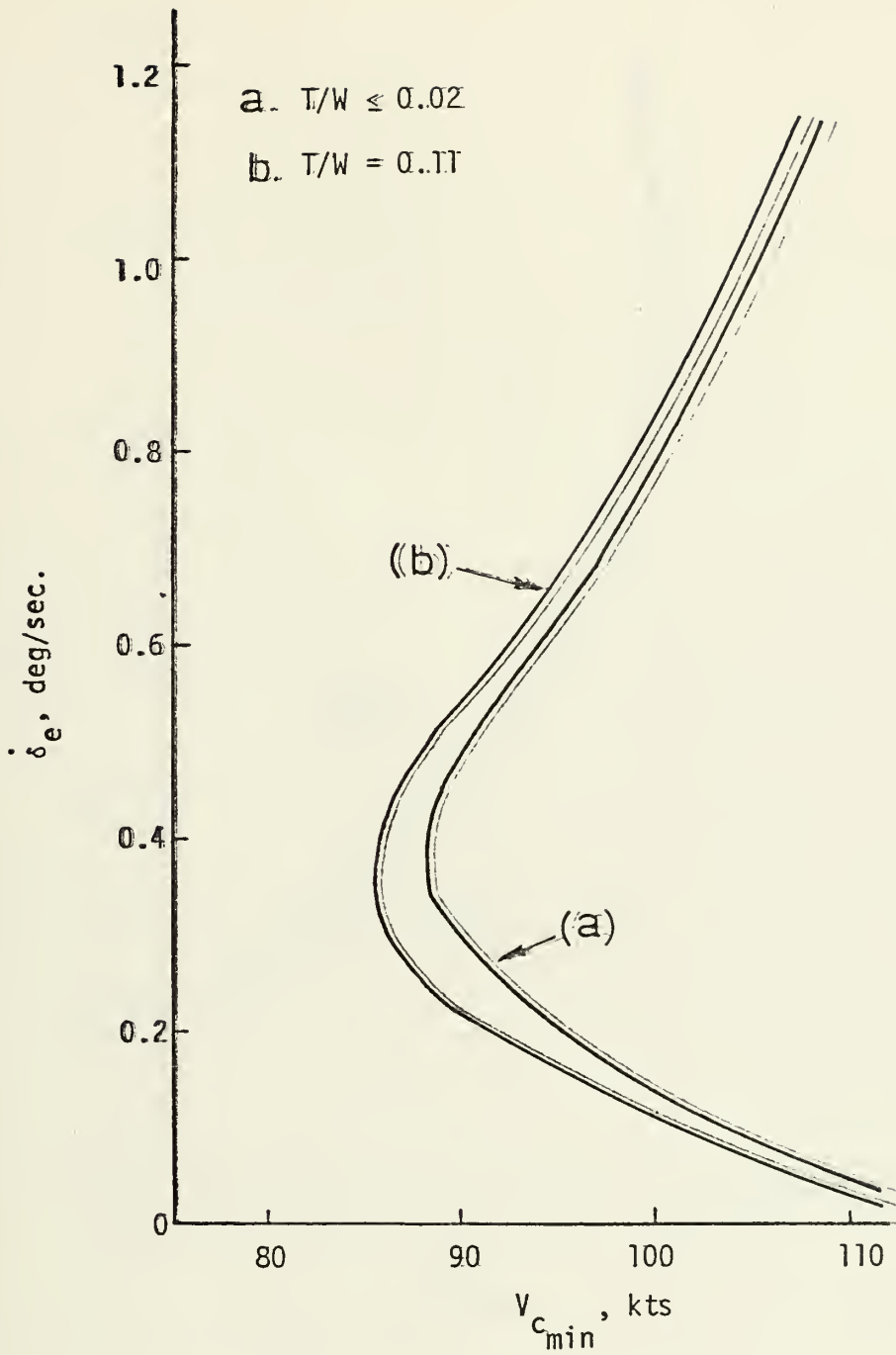


Figure 9. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

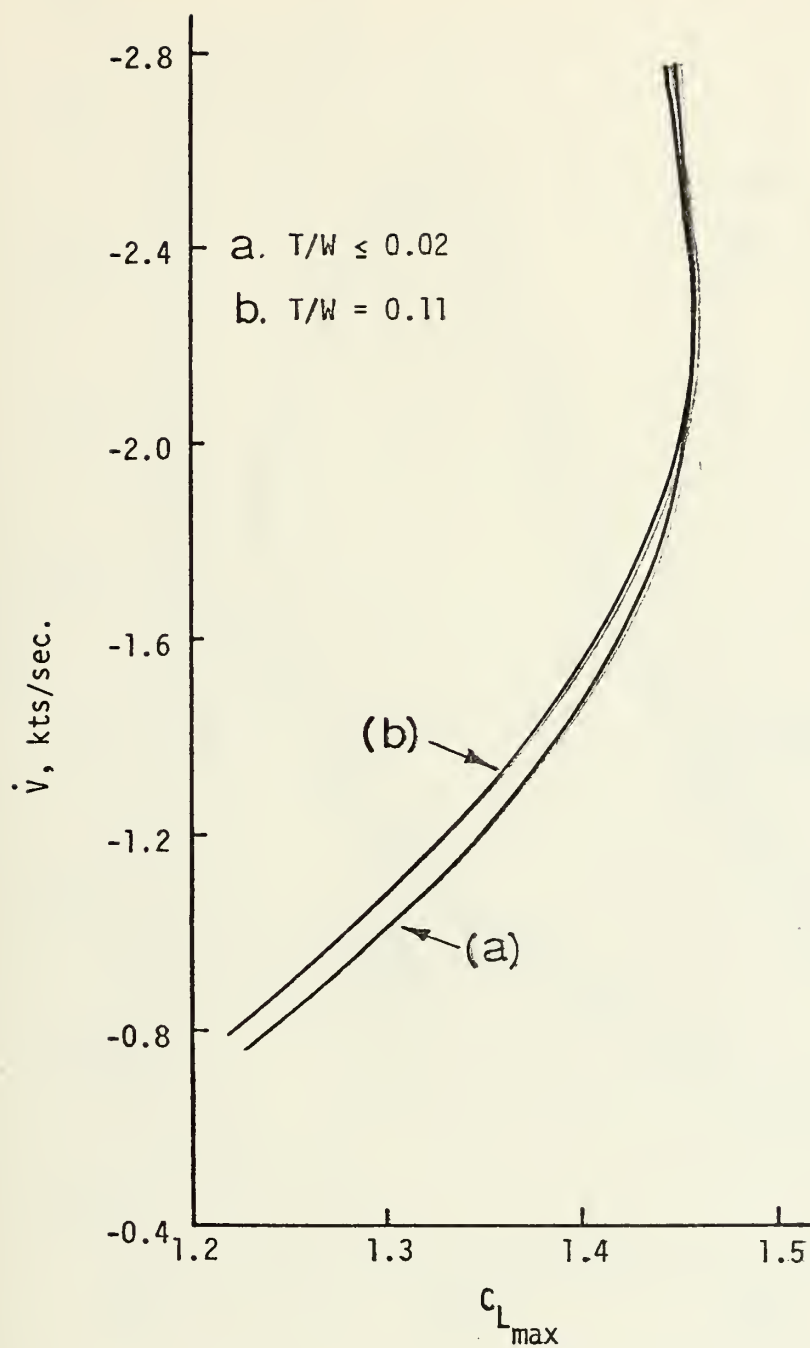


Figure 10. True Maximum Lift Coefficient as a Function of Deceleration Rate.

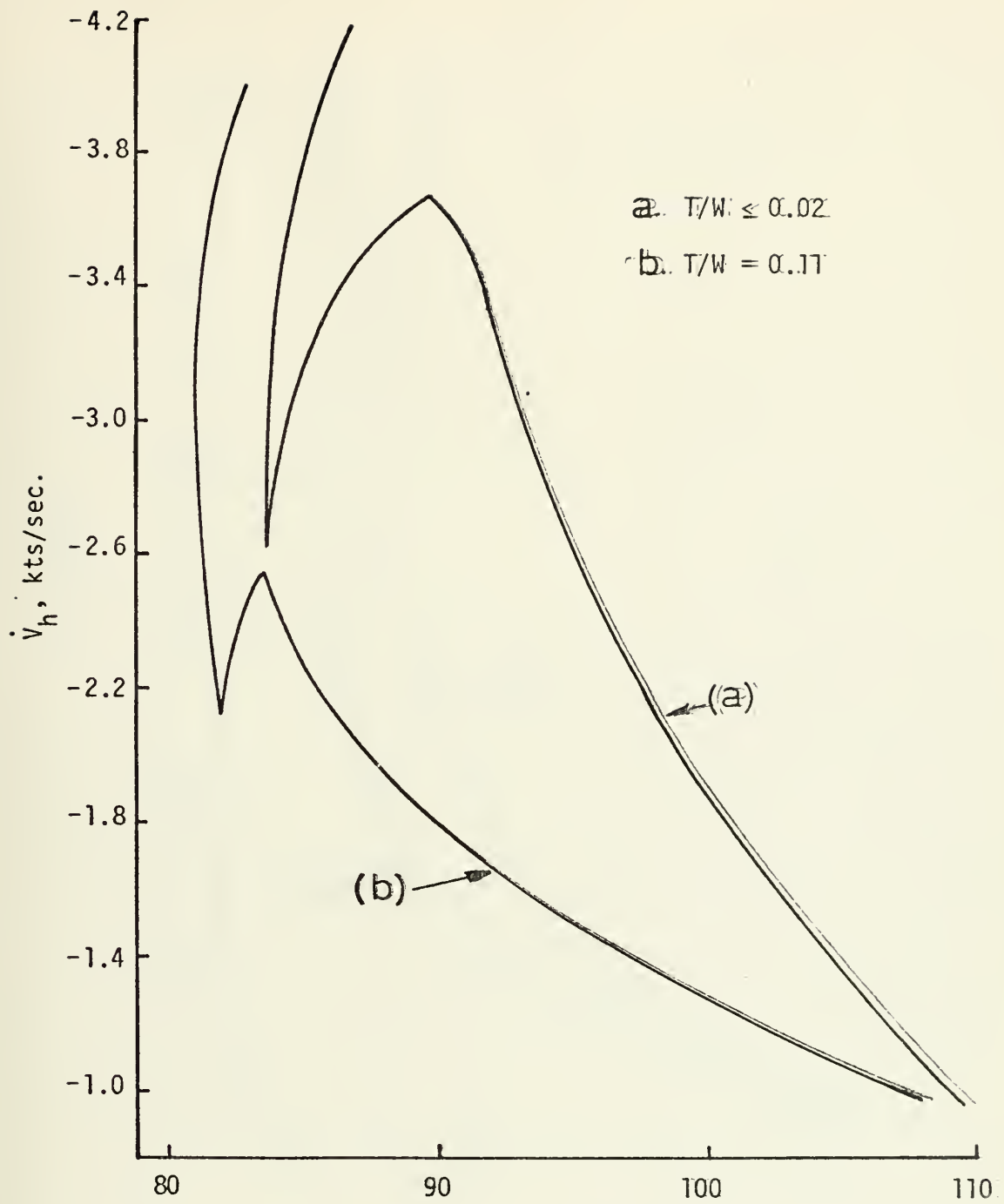


Figure 11. Minimum Flying Speed as a Function of Deceleration Rate.

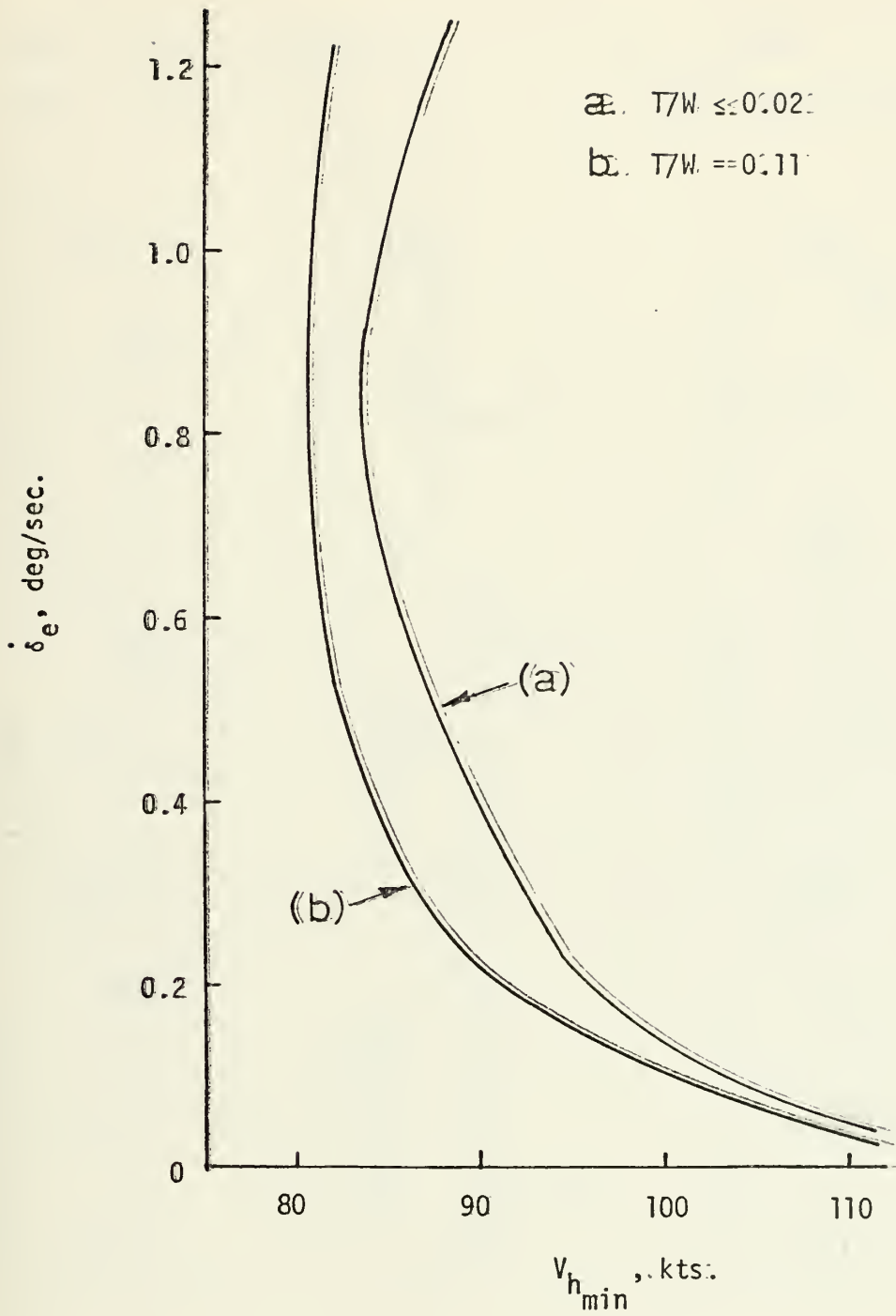


Figure 12. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

\odot \dot{V} versus V_{\min} Method
 \square C_L' Method

\triangle C_L Method
 \diamond Altitude Break Method

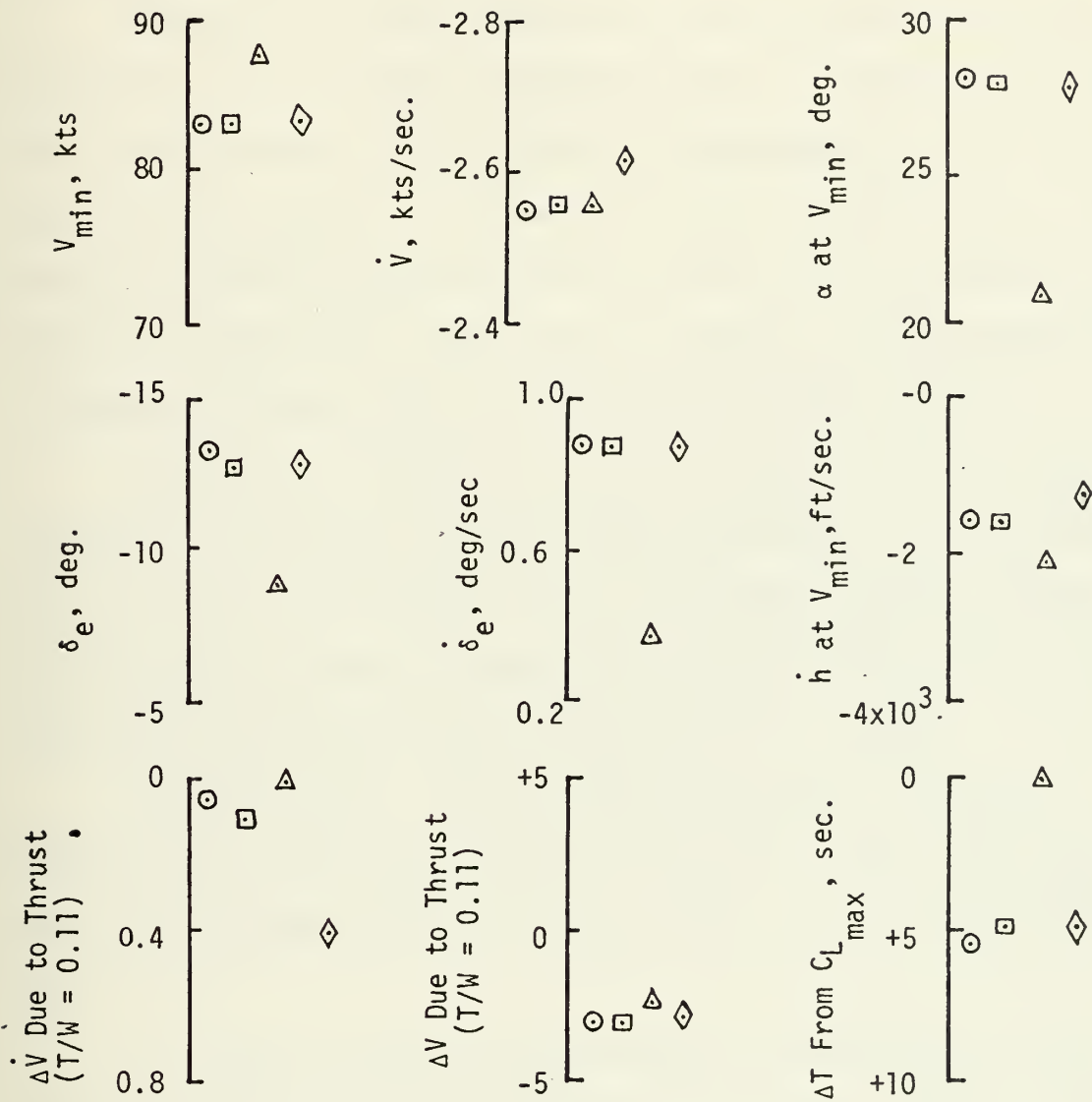


Figure 13. Comparison of Aircraft Parameters at Absolute Minimum Flying Speed $T/W \leq 0.02$.

The deceleration rate was approximately 2.6 knots/sec at the absolute minimum flying speed for all four methods. Significant changes in deceleration rate due to increased thrust was observed only in the altitude break method.

Minimum flying speed determined by the C_L method occurred at the smallest rate of change of elevator angle, -0.4 degs/sec. Minimum flying speed determined by the constant deceleration, C_L^1 and altitude break methods occurred at -0.9 deg/sec. rate of change of elevator angle. Absolute minimum flying speed defined by the C_L method occurred at the smallest elevator angle, approximately -9 degrees, whereas approximately -13 degrees elevator angle was required for the other methods.

The angle of attack at the absolute minimum flying speed was approximately 28 degrees for the constant deceleration, C_L^1 and altitude break methods, and considerably less at 21 degrees for the C_L method. Thrust effects on angle of attack at the absolute minimum flying speed were insignificant (less than 0.5 degrees) for all methods.

Elevator angles, lift coefficients and angles of attack at minimum flying speed/stall speed were less than, in some cases, the maximum static values defined for the aircraft. Possible explanations are:

1. Rate of change of elevator angle was programmed to become zero two seconds after V_{min} was obtained. For small values of elevator rate, V_{min} occurred much sooner than the maximum obtained values of C_L , α and δ_e . Therefore, elevator angle was held constant until the end of the maneuver, restricting the aircraft from obtaining the maximum values of C_L , α and δ_e .

2. The variations of drag coefficient with angles of attack used in the table look up were representative values and not necessarily true values for the F-94A. Reduced drag coefficients would therefore effect the occurrence of minimum flying speed.

3. In defense of the low values of C_L , α and δ_e , it must be remembered that this is a dynamic study of stall traits. Actual flight test data, [Ref. 4], indicates angle of attack is less than static stall angle of attack.

B. FAR MINIMUM FLYING SPEED

The minimum flying speed defined by Federal Aviation Regulations corresponds to the minimum speed obtained at a one knot/sec. deceleration rate. The FAR minimum flying speed determined by the constant deceleration, C_L and C_L' methods was approximately 105 knots. The altitude break method's FAR minimum flying speed was 109 knots. Increased thrust decreased the altitude break minimum speed two knots and decreased the minimum speed defined by the remaining methods less than one knot.

The rate of change of elevator was - 0.08 deg/sec. at minimum flying speed determined by the constant deceleration rate, C_L and C_L' methods. The altitude break method rate of change of elevator angle was -0.05 deg/sec.

Angle of attack was approximately 15 degrees and elevator angle -4.8 degrees for all methods.

C. COMPARISON OF MINIMUM FLYING SPEEDS

The absolute minimum flying speeds were categorically 20 knots less than those defined by FAR one knot/sec. deceleration rate, Figure 14. Thrust had more effect on the absolute minimum flying speed whereas rate

○ \dot{V} versus V_{\min} Method

□ C_L' Method

△ C_L Method

◇ Altitude Break Method

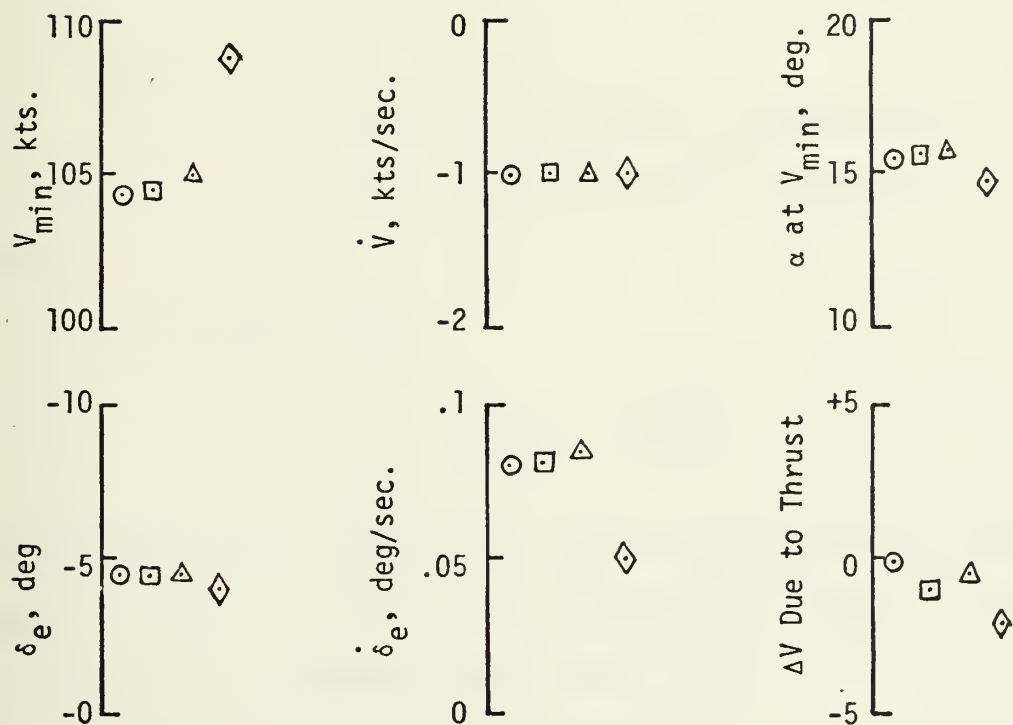


Figure 14. Comparison of Aircraft Parameters at FAR Minimum Flying Speed.

of change of elevator angle had more effect on the FAR minimum flying speeds. Angle of attack and elevator angle were significantly greater with the absolute minimum flying speeds.

D. SENSITIVITY STUDIES

1. Aircraft Weight

For a representative rate of change of elevator angle, Figure 15 indicates minimum flying speed was a linear function of aircraft weight.

From the definition:

$$C_{L_{\max}} = \frac{W}{\bar{q}_s S}$$

minimum flying speed is seen to be a function of the one-half power of weight. Actual flight test data indicates minimum flying speed is a linear function of aircraft weight [Ref. 4].

2. Aircraft Pitch Moment of Inertia

When the aircraft pitch moment of inertia was increased or decreased 25 percent, there were no significant effects on minimum flying speeds or other aircraft parameters. This conclusion was not based on a systematic study but rather a brief look at three data runs. The trends were present but a complete study is warranted.

3. Exponential Elevator Rate

Comparative plots of linear and linear plus exponential elevator rates were made in Figure 16 for initial linear rates of -0.06 and -0.115 deg/sec. The exponential rate was e^x where x was increased 0.04 each second. V_{\min} obtained by the linear plus exponential elevator rate was significantly less than that obtained by the linear elevator rate.

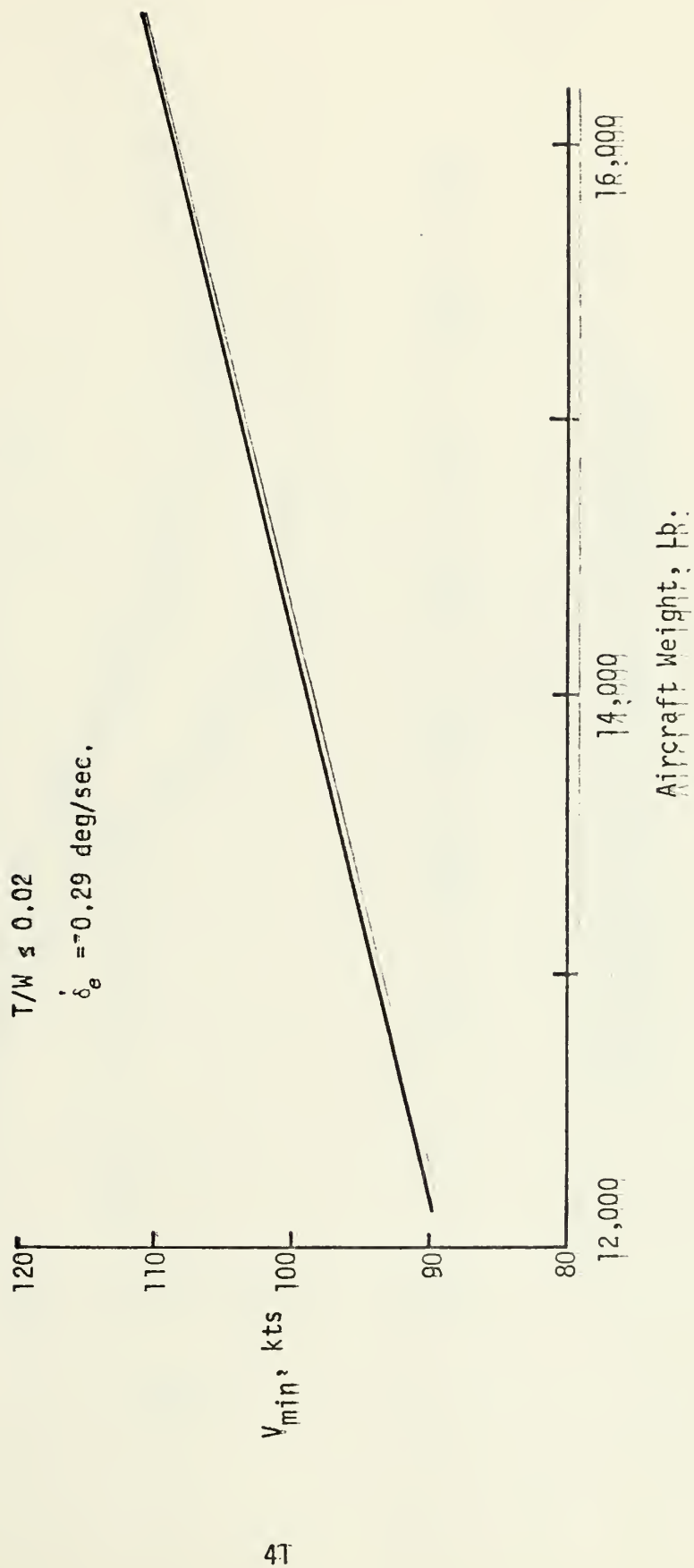


Figure 15. Minimum Flying Speed as a Function of Aircraft Weight.

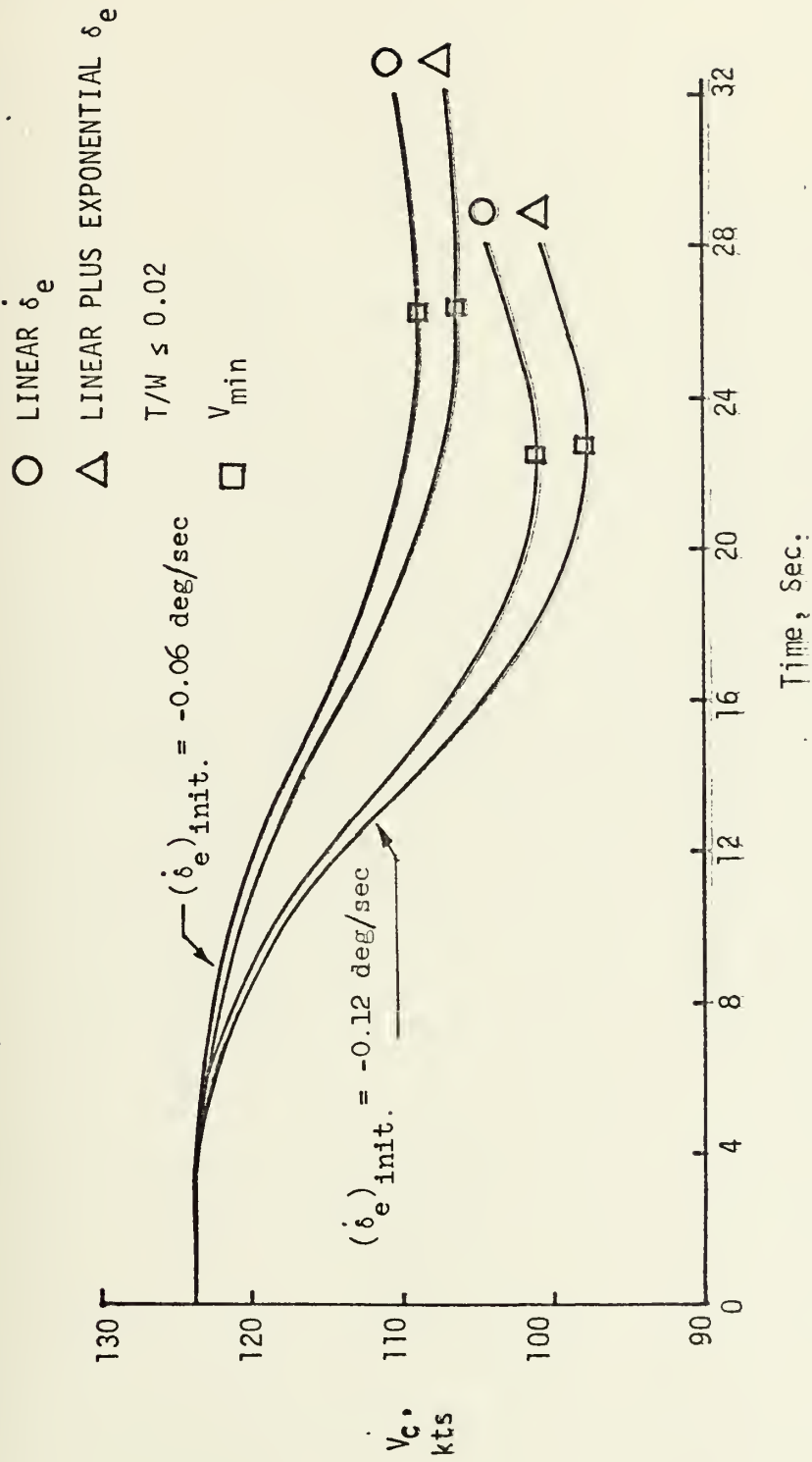


Figure 16. Comparison of Aircraft Velocity Time History for Linear and Linear Plus Exponential Rate of Change of Elevator Angle.

These results, though only a sampling of data, tend to support the existence of an optimum elevator schedule in determining minimum flying speed.

IV. CONCLUSIONS AND RECOMMENDATIONS

Digital simulation has great potential in the evaluation of minimum flying speed. The options available are practically unlimited.

The determination of absolute minimum flying speed and aircraft behavior with digital simulation will enable the engineer to better predict the aircraft characteristics encountered during test flight. The test pilot's job will be simplified somewhat if he has knowledge of general aircraft characteristics prior to flight.

The operational pilot's performance should be enhanced by having an insight into his aircraft's performance beyond the certified stall speed.

The computer program coded for this study may be used for many fields of study as indicated by Ref. 7, where it was used to make a stability analysis in the vicinity of minimum flying speed.

APPENDIX A

NON-LINEAR EQUATIONS OF AIRCRAFT MOTION

I. Euler's Equations of Aircraft Motion.

The aircraft was considered as a rigid body with a Cartesian coordinate system fixed at the center of mass, Figures 17. The frame of reference Cxyz, a body axis system, moved with the aircraft and CX was fixed to the longitudinal axis of the aircraft. The resultant external forces and the moments of these forces about the aircraft center of mass, when referred to the frame of reference Cxyz, where the vector equations of motion:

$$\overline{F} = m \frac{d\overline{V}_C}{dt} + m \overline{\omega} \times \overline{V}_C$$

$$\overline{G} = \frac{d\overline{h}}{dt} + \overline{\omega} \times \overline{h}$$

The scalar components of these equations are Euler's equations of aircraft motion.

II. Aircraft Orientation and Axes.

Reference frame Cxyz could not be used to describe the position and orientation of the aircraft as it was fixed to the center of mass and moved with the aircraft. An earth fixed reference frame, assuming the earth's rotation was negligible, was chosen to describe the aircraft motion.

Euler's equations of motion are valid for any orthogonal reference frame fixed to the aircraft center of mass. To simplify the equations of motion and expressions for aerodynamic forces, the reference frame,

Cx'y'z', was chosen as stability axes, i.e., Cx' pointed in the direction of motion of the aircraft center of mass, Figure 17.

III. Assumptions

In addition to the assumptions previously made defining aircraft orientation and axes it was assumed that aircraft motion was restricted to the Cxy, longitudinal plane. This assumption equated the following quantities to zero: L, N, P, R, Φ , Ψ , V and Y. Cxy was also assumed to be a plane of symmetry which equated the products of inertia, D and F, to zero.

IV. Non-Dimensionalization

The external aerodynamic forces and moments were expressed as functions of dimensionless force and moment coefficients, dynamic pressure, aircraft planform wing area and a reference length. These equations, in scalar form, were:

$$X = C_x \frac{1}{2} \rho V_c^2 S$$

$$Z = C_z \frac{1}{2} \rho V_c^2 S$$

$$M = C_m \frac{1}{2} \rho V_c^2 S \bar{c}$$

When these values were substituted into the scalar resultant external force and moment equations the following equations were obtained:

$$\dot{U} = \frac{\rho V_c^2 S C_x}{2m} - g \sin \theta - QW$$

$$\dot{W} = \frac{\rho V_c^2 S C_z}{2m} + g \cos \theta + QU$$

—— BODY AXES
- - - STABILITY AXES

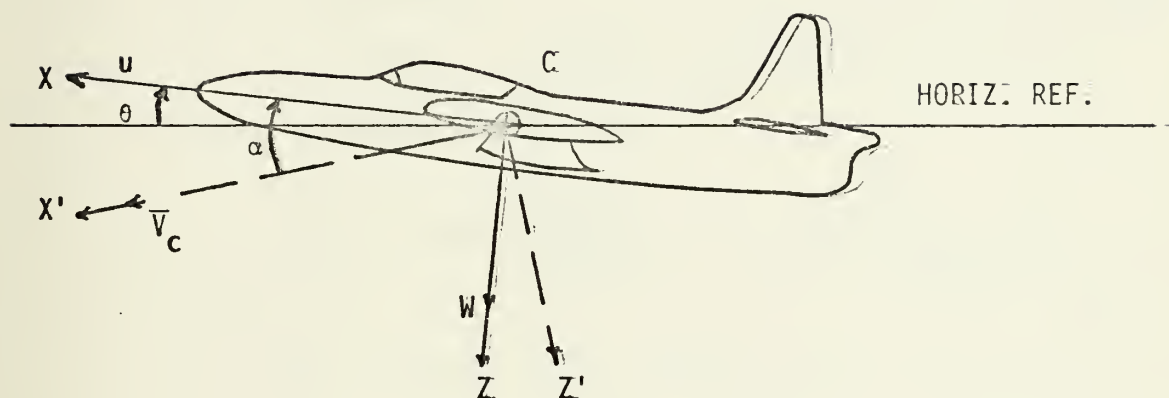


Figure 17. Aircraft Axes.

$$\dot{Q} = \frac{\rho V_c^2 S \bar{c} C_m}{B}$$

$$Q = \dot{\theta}$$

V. Aerodynamic Force and Moment Coefficients

From Figure 18, the force coefficients were:

$$C_x = C_T + C_L \sin \alpha - C_D \cos \alpha$$

$$C_z = -C_L \cos \alpha - C_D \sin \alpha$$

The pitching moment and lift coefficients consisted of linear portions. The non-linearity was present in $C_L(\alpha)$ and $C_m(\alpha)$, which were non-linear curves of C_L and C_m versus α . Complete expressions for the total pitching moment and lift coefficients were:

$$C_m = C_m(\alpha) + \frac{c}{2V_c} \left[C_{m_q} \dot{\theta} + C_{m_{\dot{\alpha}}} \dot{\alpha} \right] + C_{m_{\delta e}} \delta \dot{e}$$

$$C_L = C_L(\alpha) + C_{L_{\dot{\theta}}} \dot{\theta} + C_{L_{\delta e}} \delta \dot{e}$$

VI. Final Equations

Angle of attack appeared in the pitching moment equation as a derivative. To obtain a set of simultaneous differential equations that could be solved by one of the methods available, another equation was added defining the rate of change of angle of attack. The three-degrees-of-freedom, non-linear differential equations of aircraft motion programmed for the Naval Postgraduate School IBM 360 digital computer were:

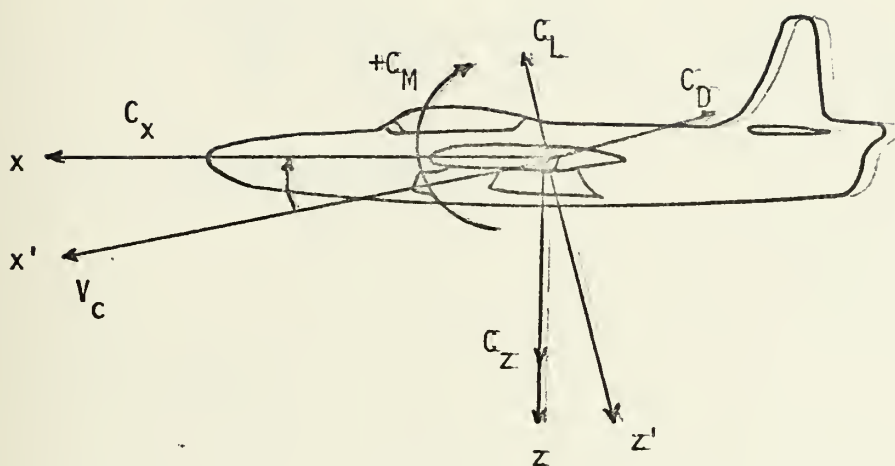


Figure 18. Aerodynamic Force and Moment Coefficients.

$$\dot{U} = \frac{\rho V_c^2 S}{2m} \left[C_T + \left(C_L(\alpha) + C_{L\dot{\theta}} \ddot{\theta} + C_{L\delta e} \delta \dot{e} \right) \sin \alpha - C_D \cos \alpha \right] - g \sin \theta - QW$$

$$\dot{W} = \frac{\rho V_c^2 S}{2m} \left\{ - \left[\left(C_L(\alpha) + C_{L\dot{\theta}} \ddot{\theta} + C_{L\delta e} \delta \dot{e} \right) \cos \alpha + C_D \sin \alpha \right] \right\} + g \cos \theta + QU$$

$$\dot{\theta} = Q$$

$$\dot{Q} = \frac{\rho V_c^2 S \bar{c}}{2B} \left[C_m(\alpha) + \frac{c}{2V} \left(C_{m\dot{\alpha}} \ddot{\alpha} + C_{m\dot{q}} \dot{q} \right) + C_{m\delta e} \delta \dot{e} \right]$$

$$\dot{\alpha} = \frac{\dot{W} \cos \alpha - \ddot{U} \sin \alpha}{U}$$

APPENDIX B

DEVELOPMENT OF COMPUTER PROGRAM

I. COMPUTER FACILITY

The original intent was to program these equations of motion on the Aeronautical Engineering Department EAI 580 analog computer. Problems with insufficient equipment to program these equations of motion on the analog computer necessitated the use of digital simulation.

The Naval Postgraduate School's IBM 360 digital computer offered many advantages and was chosen as the research tool. Several major advantages were: the large amount of core available; dimensioned quantities could be used; plot subroutines were available for time history studies and numerous methods were available for solving simultaneous non-linear differential equations on a digital computer.

II. COMPUTER PROGRAM

Of the many one step procedures that were available for solving systems of simultaneous first-order ordinary differential equations, a fourth-order Runge-Kutta algorithm was chosen, [Ref. 8]. This method offered simplicity, accuracy and short computer time. The Runge-Kutta procedures were programmed into subroutine RUNKUT. RUNKUT called function subroutines F1, F2, F3, F4 and F5 to calculate the derivative terms at time $T(I)$, $T(I)+HI/2$ and $T(I)+HI$. The derivative terms were multiplied by the appropriate weight factors and added to the variables calculated at time $T(I)$ which gave values for the variables at time $T(I+1)$.

Function subroutine FT, called subroutine SPLIN to calculate the non-linearities in C_L , C_m and C_D . SPLIN was obtained from the scientific subroutine package of the Naval Postgraduate School IBM 360 digital computer and modified for this program. Non-linear tables of C_L , C_m and C_D as a function of angle of attack were read into SPLIN. When supplied values of angle of attack, SPLIN used a cubic spline function to provide interpolated values of C_L , C_m and C_D .

Initial aircraft trim conditions U , W , θ , $\dot{\theta}$, α and δ_e were supplied to RUNKUT at time zero for initiation of the step procedure. These values were calculated in Subroutine TRIM from static force and moment equations.

Control of the aircraft was accomplished through changes in elevator angle and thrust which simulated control stick and throttle movements. The options available were selected through the integers on the first data card.

III. TRUNCATION ERROR AND STEP-SIZE ANALYSIS

Error analysis by rigorous mathematical derivations are virtually impossible to implement for higher-order Runge-Kutta algorithms for systems of differential equations. The truncation error:

$$e_t = \frac{16}{15} (y_{n+1,2} - y_{n+1,1})$$

for a first-order ordinary differential equation was applied to the system of differential equations as a step-size control mechanism, [Ref. 8]. Step-size and truncation errors are listed in Table II.

Table II

Local Truncation Errors

<u>Step-Size</u>	<u>Error</u>
0.2	-2.92×10^{-4}
0.1	-3.97×10^{-5}
0.05	11.56×10^{-6}
0.025	-3.9×10^{-8}

PROGRAMMED BY LT. C. W. SAUL TO OPERATE ON THE NAVAL
PDCSTGRADUATE IBM 360 DIGITAL COMPUTER. PROGRAMED IN
THESES, "EVALUATION OF MINIMUM
AIRCRAFT FLYING SPEED BY DIGITAL SIMULATION", FOR PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR A MASTERS DEGREE IN
AERONAUTICAL ENGINEERING.

THE PROGRAM SOLVES THE NON-LINEAR, THREE-DEGREES-OF-FREEDOM EQUATIONS OF AIRCRAFT MOTION BY USING A FOURTH-ORDER RUNGE-KUTTA ALGORITHM.

THE PROGRAM WAS WRITTEN TO PROVIDE NUMEROUS OPTIONS IN THRUST AND RATES OF CHANGE OF ELEVATOR ANGLE (CONTROLS THE AIRCRAFT BY STICK AND THROTTLE MOVEMENTS). THE OPTIONS WERE INCORPORATED TO BE CONTROLLED BY FOUR INTEGERS: I1, I2, I3 AND I4, LISTED ON THE FIRST DATA CARD. BY CHANGING ONLY THE FIRST DATA CARD, THE VARIOUS OPTIONS IN THRUST AND RATE OF CHANGE OF ELEVATOR ANGLE CAN BE CHOSEN. VALUES FOR THE INTEGERS ARE PLACED IN THE FIRST EIGHT COLUMNS OF THE DATA CARD WITH TWO SPACES ALLOWED FOR EACH VALUE. THE VALUES ARE RIGHT JUSTIFIED. THE OPTIONS AVAILABLE ARE:

II THRUST OPTION

VALUE

OPTION

1. THRUST WILL BE CALCULATED FOR INPUT ANGLE OF ATTACK AND PITCH ANGLE BY SUBROUTINE TRIM AND WILL REMAIN CONSTANT THROUGHOUT THE FLIGHT

2 AIRCRAFT WILL BE TRIMMED AND FLOWN WITH ZERO THRUST

3 TRIM THRUST WILL BE THE SAME AS FOR $I_1=1$ BUT WILL BE GRADUALLY REDUCED TO A $T/W=0.02$ DURING THE FIRST 1.2 SECONDS OF FLIGHT

4. THE AIRCRAFT WILL BE TRIMMED AND FLOWN FOR $T/W=9.02$

112 ELEVATOR SCHEDULE (13 MUST EQUAL 1)

DATE	DESCRIPTION	AMOUNT	CHECK NO.	BANK	VALUE
10/1/20	DEPOSIT	100.00			
10/5/20	PAYROLL	50.00	101	ABC BANK	
10/10/20	RENT	200.00	102	ABC BANK	
10/15/20	SALES	150.00			
10/20/20	UTILITIES	30.00	103	ABC BANK	
10/25/20	SALES	120.00			
10/30/20	SALES	80.00			
11/5/20	SALES	90.00			
11/10/20	SALES	110.00			
11/15/20	SALES	130.00			
11/20/20	SALES	140.00			
11/25/20	SALES	160.00			
11/30/20	SALES	180.00			
12/5/20	SALES	200.00			
12/10/20	SALES	220.00			
12/15/20	SALES	240.00			
12/20/20	SALES	260.00			
12/25/20	SALES	280.00			
12/30/20	SALES	300.00			
TOTAL		2000.00			

OPTION

TO TRIM THRUST (I1=3) OR INCREASED BY (TRIM THRUST-
250 LB.) OVER A 1.2 SECOND PERIOD

EXPONENTIAL ELEVATOR SCHEDULE:

FOR THE EXPONENTIAL ELEVATOR SCHEDULE I3=2 AND VALUES FOR
I2=1 THRU 9 ARE USED. WHEN THE AIRCRAFT DECELERATION RATE
FALLS BELOW THE DESIRED RATE (DENOTED AS NUM1 AND IN KTS/SEC)
THE PROGRAM AUTOMATICALLY SWITCHES TO AN EXPONENTIAL
ELEVATOR SCHEDULE. THE ORIGINALLY SELECTED LINEAR SCHEDULE
IS MULTIPLIED BY E RAISED TO THE RI POWER. THE LINEAR VALUES
ARE THE SAME AS DEFINED BY I2. THE DESIRED EXPONENTIAL
RATE (RI) MUST BE INSERTED INTO THE PROGRAM PRIOR TO
EXECUTION.

IMPLICIT REAL*8(A-H,K-Z)
REAL*4 RANGE,RANGE1,RANGE2,RANGE3,RANGE4,RANGE5,RANGE6

THE FUNCTION SUBROUTINES UTILIZED BY SUBROUTINE RUNKUT ARE
DEFINED AS:

FUNCTION F1 = U-DOT
FUNCTION F2 = W-DOT
FUNCTION F3 = THETA-DOT
FUNCTION F4 = THETA-DBL. DOT
FUNCTION F5 = ALPHA-DOT

THE FOLLOWING DIMENSIONED QUANTITIES ARE:

T(I)=REAL TIME
X1(I)=U VELOCITY COMPONENT
X2(I)=W VELOCITY COMPONENT
X3(I)=PITCH ANGLE THETA IN RADIANS
X4(I)=RATE OF CHANGE OF PITCH ANGLE (THETA DOT IN RADIANS)
X5(I)=ANGLE OF ATTACK ALPHA IN RADIANS
H(I)=ALTITUDE IN FEET
HDOT(I)=RATE OF CHANGE ALTITUDE IN FT. PER MINUTE
ANF(I)=FLIGHT PATH NORMAL ACCELERATION
ACL(I)=AIRCRAFT LIFT COEFFICIENT
ACD(I)=AIRCRAFT DRAG COEFFICIENT


```

ACM(I)=AIRCRAFT PITCHING MOMENT COEFFICIENT
DEL(I)=ELEVATOR ANGLE
CLP(I)=APPARENT AIRCRAFT LIFT COEFFICIENT
VEL(I)=AIRCRAFT RESULTANT VELOCITY

DIMENSION T(641),X1(641),X2(641),X3(641),X4(641),X5(641),VEL(641),
1ANFP(641),H(641),ACD(641),ACM(641),DEL(641),CLP(641),RANG
2E(4),RANGE1(4),RANGE2(4),RANGE3(4),RANGE4(4),RANGE5(4),RANGE6(4)

COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAC,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM

READ(5,37) I1,I2,I3,I4
READ(5,1) TT,CLTD,CLDE,RHO,S,MA,JM
READ(5,2) CMTD,CMAD,CMDE,BIY,CBAR

```

THE FOLLOWING ARRAYS ARE READ INTO THE PROGRAM TO BE USED BY SUBROUTINE SPLIN1. SPLIN1 IS GIVEN 14 SETS VALUES TO BE USED AS ORIGINATE AND ABASSIA FOR A TABLE LOOK UP. SPLIN1 TAKES A GIVEN VALUE OF ANGLE OF ATTACK AND PROVIDES THE NON-LINEARITY IN AIRCRAFT CL, CD AND CM. THE ARRAYS ARE:

```

X=VALUES OF ANGLE OF ATTACK
Y=VALUES OF LIFT COEFFICIENTS
V=VALUES OF DRAG COEFFICIENTS
XX=VALUES OF PITCHING MOMENT COEFFICIENT

```

SPLIN1 TAKES A GIVEN VALUE OF ANGLE OF ATTACK AND PROVIDES THE CORRESPONDING VALUE OF CL,CD OR CM

```

READ(5,3) X
READ(5,3) Y
READ(5,3) V
READ(5,3) XX

```

STEP SIZE HI USED IN RUNGA-KUTTA SCHEME IS IN REAL TIME SECONDS. JJ, JJJ, IJ, JI ARE INTEGERS USED IN PROGRAMING THE DESIRED RATE OF CHANGE OF ELEVATOR ANGLE.

```

HI=0.5D-01
G=32.174
JJ=1

```



```

JJJ=3
NUM1=2.15
R1=0.0
IM=641
INP=IM-1
IJ=40
II=1
JI=1

```

THE DENSITY ALTITUDE IS SEA LEVEL, BUT THE AIRCRAFT WAS
ARBITRARILY PLACED AT 5M FEET TO AVOID NEGATIVE ALTITUDES.

```

H(1)=5000

```

THE FOLLOWING VALUES OF RANGE DEFINE THE SCALES FOR THE FOUR
OUTPUT GRAPHS.

```

RANGE(1)=32.0
RANGE(2)=0.0
RANGE(3)=129.0
RANGE(4)=87.0
RANGE1(1)=32.0
RANGE1(2)=0.0
RANGE1(3)=7000.0
RANGE1(4)=1000.0
RANGE2(1)=32.0
RANGE2(2)=0.0
RANGE2(3)=2.0
RANGE2(4)=0.8
RANGE3(1)=32.0
RANGE3(2)=0.0
RANGE3(3)=2.0
RANGE3(4)=0.8
RANGE4(1)=32.0
RANGE4(2)=0.0
RANGE4(3)=32.0
RANGE4(4)=2.0
RANGE5(1)=32.0
RANGE5(2)=0.0
RANGE5(3)=18.0
RANGE5(4)=0.0
RANGE6(1)=32.0
RANGE6(2)=0.0
RANGE6(3)=1.6

```

CCCCC CCCCCC

RANGE6(4)=0.4

THE INITIAL CONDITIONS ARE READ INTO THE PROGRAM.

T(1)=0.0D0
READ(5,4) W,AOA,THETA

ALL VALUES READ INTO THE PROGRAM ARE PRINTED OUT AS AN ECHO CHECK.

```
WRITE(6,31)
WRITE(6,5) TT,CLTD,CLDE,RHO,S,MA,JM,CMTD,CMAD,CMDE,BIY,CBAR,II,
1I2,I3,I4
WRITE(6,6)
WRITE(6,7) (X(I),Y(I),V(I),XX(I), I=1,14)
ATH=THETA-ACA
IF(CABS(ATH).GT.0.1D-02) GO TO 9
WRITE(6,8) AOA,W
```

THE INPUT DATA IS USED TO CALCULATE THE TRIM CONDITIONS FOR STEADY LEVEL FLIGHT. SUBROUTINE TRIM TAKES ANGLE OF ATTACK PLUS OTHER AIRCRAFT DATA AND CALCULATES THE CORRESPONDING VELOCITY, ELEVATOR ANGLE, THRUST, CL, CD, AND CM. SUBROUTINE TRIM USES THE FORCE AND MOMENT EQUATIONS AT STATIC EQUILIBRIUM. THIS CALCULATOR DATA IS THEN USED AS THE INITIAL CONDITIONS TO START THE RUNKALUTTA ITERATIVE SCHEME.

```
CALL TRIM(W,THETA,AOA,B1,C1,D1,E1,FF,VLL,II)
WRITE(6,17)
WRITE(6,18) B1,C1,VLL,D1,E1,FF,DE,CL,CM,CD,TT
X1(1)=B1
X2(1)=C1
X3(1)=D1
X4(1)=E1
X5(1)=FF
DEL(1)=DE
ACL(1)=CL
ACM(1)=CM
ACD(1)=CD
A=T(1)
B=X1(1)
```



```

C=X2(1)
D=X3(1)
E=X4(1)
F=X5(1)
WRITE(6,21)

```

THE PROGRAM NOW PROVIDES THE DESIRED THRUST OPTION.

```

DELT=(TT-250.0)/24.0
IF(DELT) 101,101,102
DELT=0.0
DO 20 I=1, IMM
  IF(I.GE.25) GO TO 14
  GO TO (14,12,13,19), I
  TT=0.0
  GO TO 14
  TT=TT-DELT
  IF(I4.EQ.1) GO TO 19
  IF(I4.LE.80) GO TO 19
  BVEL=DSQRT(X1(I-1)*X1(I-1)+X2(I-1)*X2(I-1))
  CVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
  XVEL=BVEL-VEL
  IF(XVEL) 15,19,19
  IF(I4.GE.3) GO TO 16
  I4=I+60
  IF(I.LT.14) GO TO 19
  IF(I4.GE.25) GO TO 19
  TT=TT+DELT
  JJ=JJ+1

```

SUBROUTINE RUNKUT USES THE TRIM VALUES CALCULATED BY THE SUBROUTINE TRIM AS INITIAL CONDITIONS TO START THE ALGORITHM. THESE VALUES ARE THEN USED AS INITIAL CONDITIONS IN THE CALCULATING THE AIRCRAFT PARAMETERS AT THE NEXT TIME STEP. THEREFORE, ONCE STARTED, THE RUNGE-KUTTA ALGORITHM IS SELF SUSTAINING.

```

CALL RUNKUT(A,B,C,D,E,F,BI,C1,D1,E1,FF)
X1(I+1)=BI
X2(I+1)=CI
X3(I+1)=DI
X4(I+1)=EI
X5(I+1)=FF

```



```

T(I+1)=T(I)+HI
A=T(I+1)
B=X1(I+1)
C=X2(I+1)
D=X3(I+1)
E=X4(I+1)
F=X5(I+1)
DEL(I+1)=DE
ACL(I+1)=CL
ACM(I+1)=CM
ACD(I+1)=CD

```

THE PROGRAM NOW PROVIDES THE DESIRED ELEVATOR SCHEDULE THAT IS CHOSEN.

```

IF(DABS(DEL(I)),GE,0.28) GO TO 20
IF(I3.GE.2) GO TO 57
GO TO (23,24,24,20,25,25,25), I2
DE=DE-(0.1D-03*I2)
GO TO 20
DE=DE-(0.15D-03+(I2-3)*(0.1D-03))
GO TO 20
IF(I1.GE.IJ) GO TO 20
IF(I1.LE.40) GO TO 54
BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
CVEL=DSQRT(X1(I+1)*X1(I+1)+X2(I+1)*X2(I+1))
XVEL=BVEL-CVEL
IF(XVEL) 51,51,54
II=II+1
IF(I2-8) 55,56,56
DE=DE-(0.1D-03*(I2-5))
GO TO 20
DE=DE-(0.15D-03+(I2-8)*(0.1D-03))
GO TO 20
GO TO (20,80,80,58), I3
IF(I1.GE.IJ) GO TO 59
IF(I1.LE.40) GO TO 54
BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
CVEL=DSQRT(X1(I+1)*X1(I+1)+X2(I+1)*X2(I+1))
XVEL=BVEL-CVEL
IF(XVEL) 51,54,54
DE=DE+0.1D-03
IF(DEL(I)-DE) 60,60,20
DE=DEL(I)
GO TO 20
IF(J1.GE.IJ) GO TO 20

```

CCCCC

23

24

25

51

54

55

56

57

58

59

60

80


```

IF(I.LE.40) GO TO 86
BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
CVEL=DSQRT(B1*B1+C1*C1)
XVEL=BVEL-CVEL
IF(XVEL) 81,81,82
J1=J1+1
DVEL=(XVEL/0.05)*0.592086
IF(DVEL-NUM1) 83,86,86
GO TO (20,84,85,58), I3
R1=R1+0.002
DE=DE-(0.1D-03*I2)*DEXP(R1)
GO TO 20
R1=R1+0.002
DE=DE-(0.15D-03+(I2-3)*(0.1D-03))*DEXP(R1)
GO TO 20
GO TO (20,23,24,58), I3
CONTINUE

```

81
82
83
84
85
86
20
C
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C

THE RESULTANT AIRCRAFT VELOCITY, NORMAL FLIGHT PATH ACCELERATION,
APPARENT LIFT COEFFICIENT AND ALTITUDE IS CALCULATED FOR EACH
TIME STEP.

```

DO 30 I=1,IM
VEL(I)=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))*0.592086
ANFP(I)=((VEL(I)*X4(I))/G)+DCOS(X3(I))-X5(I)
CLP(I)=ACL(I)/ANFP(I)
HDOT=VEL(I)*DSIN(X3(I))-X5(I)*101.3364
IF(I.EQ.IM) GO TO 30
H(I+1)=H(I)+(HDOT*(T(I+1)-T(I))/60.0)
CONTINUE
J=75

```

30

```

DO 41 I=1,IM
IF(I.EQ.J) GO TO 42
WRITE(6,22) T(I),X1(I),X2(I),VEL(I),X3(I),X4(I),X5(I),REF(I),ACT(I),CLP(I),ACM(I),ACD(I),ANFP(I),H(I))
CONTINUE
GO TO 36
WRITE(6,21)
J=J+75
GO TO 40

```

40
41
41
42

```

WRITE(6,32)
DO 50 I=1,IM
X5(I)=X5(I)*57.29577951
DEL(I)=DEL(I)*57.29577951*(-1.0)
CONTINUE
CALL UTPLOT(T,VEL,641,RANGE,2,0)

```

36
50


```

31  FORMAT('1',)
32  FORMAT('1',,///,T34,'RESULTANT AIRCRAFT VELCCITY(KTS.) VS. TIME',//
1/)
33  FCFORMAT('1',,///,T41,'AIRCRAFT ALTITUDE VS. TIME',,///)
34  FFORMAT('1',,///,T43,'AIRCRAFT CL VS. TIME',,///)
35  FFORMAT('1',,///,T30,'AIRCRAFT CL-* (BASED ON NORMAL FLIGHT PATH ACC
1EL.) VS. TIME',,///)
37  FFORMAT(4I2)
71  FFORMAT('1',,///,T36,'AIRCRAFT ANGLE OF ATTACK (DEG) VS. TIME',,///)
72  FFORMAT('1',,///,T39,'ELEVATOR ANGLE (T.E. UP) VS. TIME',,///)
73  FFORMAT('1',,///,T35,'NORMAL FLIGHT ACCELERATION VS. TIME',,///)
100 STOP
END

```

```

SUBROUTINE RUNKUT(A,B,C,D,E,F,B1,C1,D1,E1,FF)

```

SUBROUTINE RUNKUT EMPLOYES A FOURTH-ORDER RUNGE-KUTTA ALGORITHM IN SOLVING THE NON-LINEAR EQUATIONS OF AIRCRAFT MOTION. RUNKUT CALLS FUNCTION SUBROUTINES F1, F2, F3, F4 AND F5 TO CALCULATE THE DERIVATIVES IN TERMS AT INTERMEDIATE TIME STEPS. THE NON-LINEARITIES IN THE AERODYNAMIC DATA IS INCORPORATED IN F1 AND IS OBTAINED FROM TABLE LOOK-UP BY USING A CUBIC SPLINE FUNCTION; SUBROUTINE SPLINI. INPUT DATA USED BY SPLINI IN THE TABLE LOOK-UP PROCEDURE ARE LISTED IN TABLE I:

```

IMPLICIT REAL*8(A-H,K-Z)

```

```

HI=0.5D-01
KI=HI*F1(A,B,C,D,E,FF)
LI=HI*F2(A,B,C,D,E,FF)
MI=HI*F3(A,B,C,D,E,FF)
NI=HI*F4(A,B,C,D,E,FF)
PI=HI*F5(A,B,C,D,E,FF)
K1=HI*F1(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,FF+P1/2.0)
L1=HI*F2(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,FF+P1/2.0)
M1=HI*F3(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,FF+P1/2.0)
N1=HI*F4(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,FF+P1/2.0)
P1=HI*F5(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,FF+P1/2.0)
K2=HI*F1(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,FF+P2/2.0)
L2=HI*F2(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,FF+P2/2.0)
M2=HI*F3(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,FF+P2/2.0)
N2=HI*F4(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,FF+P2/2.0)
P2=HI*F5(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,FF+P2/2.0)
K3=HI*F1(A+HI/2.0,B+K3/2.0,C+L3/2.0,D+M3/2.0,E+N3/2.0,FF+P3/2.0)
L3=HI*F2(A+HI/2.0,B+K3/2.0,C+L3/2.0,D+M3/2.0,E+N3/2.0,FF+P3/2.0)
M3=HI*F3(A+HI/2.0,B+K3/2.0,C+L3/2.0,D+M3/2.0,E+N3/2.0,FF+P3/2.0)
N3=HI*F4(A+HI/2.0,B+K3/2.0,C+L3/2.0,D+M3/2.0,E+N3/2.0,FF+P3/2.0)
P3=HI*F5(A+HI/2.0,B+K3/2.0,C+L3/2.0,D+M3/2.0,E+N3/2.0,FF+P3/2.0)
K4=HI*F1(A+HI/2.0,B+K4/2.0,C+L4/2.0,D+M4/2.0,E+N4/2.0,FF+P4/2.0)
L4=HI*F2(A+HI/2.0,B+K4/2.0,C+L4/2.0,D+M4/2.0,E+N4/2.0,FF+P4/2.0)
M4=HI*F3(A+HI/2.0,B+K4/2.0,C+L4/2.0,D+M4/2.0,E+N4/2.0,FF+P4/2.0)

```

CCCCCCCCCCCC


```

N4=HI*F4(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
P4=HI*F5(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
B1=B+(K1+2.0*K2+2.0*K3+K4)/6.0
C1=C+(L1+2.0*L2+2.0*L3+L4)/6.0
D1=D+(M1+2.0*M2+2.0*M3+M4)/6.0
E1=E+(N1+2.0*N2+2.0*N3+N4)/6.0
FF=F+(P1+2.0*P2+2.0*P3+P4)/6.0
RETURN
END

```

```

FUNCTION F1(T,X1,X2,X3,X4,X5)
IMPLICIT REAL*8(A-H,K-Z)
COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
AA=(X1*X1+X2*X2)
CALL SPLIN1(X,Y,JM,X5,CLA)
CALL SPLIN1(X,X,JM,X5,CMA)
CALL SPLIN1(X,V,JM,X5,CD)
CT=(2.0*TT)/(RHO*AA*S)
CL=CLA+CLTD*0.5*CBAR*X4/DSQRT(AA)+CLDE*DE
CX=CT+CL*DSIN(X5)-CD*DCOS(X5)
CZ=(-1.0*(CL*DCOS(X5)+CD*DSIN(X5)))
F1=((RHO*AA*S)/(2.0*MA))*CX-G*DSIN(X3)-X2*X4
Z1=F1
RETURN
END

```

19

```

FUNCTION F2(T,X1,X2,X3,X4,X5)
IMPLICIT REAL*8(A-H,K-Z)
COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
F2=((RHO*AA*S)/(2.0*MA))*CZ+G*DCOS(X3)+X1*X4
Z2=F2
RETURN
END

```

19

```

FUNCTION F3(T,X1,X2,X3,X4,X5)
IMPLICIT REAL*8(A-H,K-Z)
F3=X4
RETURN
END

```

19


```

FUNCTION F4(T,X1,X2,X3,X4,X5)
IMPLICIT REAL*8(A-H,K-Z)
COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1 A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
CM=CMA+(CMTD*0.5*CBAR*X4)/DSQRT(AA)+CMAD*{(Z2*DCOS(X5)-Z1*DSIN(X5)
1)*0.5*CBAR/AA)+CMDE*DE
F4=((RHO*AA*S*CBAR)/(2.0*B IY))*CM
RETURN
END

```

10

```

FUNCTION F5(T,X1,X2,X3,X4,X5)
IMPLICIT REAL*8(A-H,K-Z)
COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1 A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
F5=((Z2*DCOS(X5)-Z1*CSIN(X5))/DSQRT(AA)
RETURN
END

```

10

```

SUBROUTINE TRIM(W,THETA,AOA,B1,C1,D1,E1,FF,VLL,I1)

```

CCCCCCCCCCCCCCCC

SUBROUTINE TRIM UTILIZES THE STATIC AIRCRAFT LONGITUDINAL MOMENT AND FORCE EQUATIONS IN TRIMMING THE AIRCRAFT IN THE DESIRED ATTITUDE. DESIRED ANGLE OF ATTACK, PITCH ANGLE AND AIRCRAFT WEIGHT ARE INPUT INTO TRIM. THE AIRCRAFT CAN BE IN A CLIMBING, LEVEL OR DESCENDING FLIGHT ATTITUDE. TRIM THEN CALCULATES TRIM VALUES FOR CL, CD, CM, ELEVATOR ANGLE AND THRUST. THESE VALUES ARE THEN PASSED TO THE MAIN PROGRAM TO BE USED AS INITIAL CONDITIONS FOR STARTING THE RUNGE-KUTTA ALGORITHM.

```

IMPLICIT REAL*8(A-H,K-Z)
COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
1 A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
X5=AOA
CM=0.0
CALL SPLINI(X,Y,JM,X5,CLA)
CALL SPLINI(X,XX,JM,X5,CMA)
CALL SPLINI(X,V,JM,X5,CD)
DE=-1.0*(CMA/CMDE)
CL=CL+CLDE*DE
AT=THETA-AOA
IF(AT) 1,2,2
AT=AOA-THETA
IF(11.EQ.2) GO TO 3

```

12


```

IF(I1.EQ.4) GO TO 4
SAT=DSIN(AT)
CAT=DCOS(AT)
SA=DSIN(AOA)
CA=DCOS(AOA)
NUM1=W*(CAT*CA-SAT*SA)
DEN1=S*(CL*CA+CD*SA)
Q=NUM1/DEN1
LD=CL/CD
NUM2=W*(LD*SAT+CAT)
DEN2=LD*CA+SA
TT=NUM2/DEN2
GO TO 7
Q=(W*DCOS(AT))/(CL*S)
TT=0.0
GO TO 7
TT=W*0.019
A=(S*S)*((CL*CL)+(CD*CD))
B=2.0*S*TT*(CL*DSIN(X5)-CD*DCOS(X5))
C=((TT*TT)-(W*W))
BB=(B/(2.0*A))
Q=-BB+DSQRT((BB*BB)-(C/A))
TA=DARCS((CL*S+C+TT*DSIN(X5))/W)
THETA=X5-TA
VX=(2.0*Q)/RHO
VLL=DSQRT(VX)
B1=VLL*DCOS(X5)
C1=VLL*DSIN(X5)
D1=THETA
EI=0.0
FF=X5
RETURN
END

```

3

4

57

```

SUBROUTINE SPLINI(X,Y,M,XINT,YINT)
IMPLICIT REAL*8 (A-H);REAL*8 (P-Z)
DIMENSION X(M),Y(M),C(4,30)
CALL SPLICO(X,Y,M,C)
K=1
ENTRY SPLINN(X,Y,M,XINT,YINT)
IF(XINT-X(1)) 70,1,2
3 GO TO 7
70 YINT=Y(1)
1 RETURN
2 IF(XINT-X(K+1)) 6,4,5
4 YINT=Y(K+1)

```



```

5 RETURN
  K=K+1
  IF(M-K) 71,71,3
71 K=M-1
  GO TO 7
6 IF(XINT-X(K))13,12,7
12 YINT=Y(K)
  RETURN
13 K=K-1
  GO TO 6
7 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))
  YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))
  RETURN
END

SUBROUTINE SPLICO(X,Y,M,C)
  IMPLICIT REAL*8 (A-H),REAL*8 (O-Z)
  DIMENSION X(M),Y(M),C(4,30),P(30),E(30),A(30,3),B(30),Z(30)
  MN=M-1
  DO 2 K=1,MN
    D(K)=X(K+1)-X(K)
    P(K)=D(K)/6
    E(K)=(Y(K+1)-Y(K))/D(K)
  DO 3 K=2,MN
    B(K)=E(K)-E(K-1)
    A(1,2)=-1,-D(1)/D(2)
    A(1,3)=D(1)/D(2)
    A(2,3)=P(2)-P(1)*A(1,3)
    A(2,2)=2*(P(1)+P(2))-P(1)*A(1,2)
    B(2)=B(2)/A(2,2)
  DO 4 K=3,MN
    A(K,2)=2*(P(K-1)+P(K))-P(K-1)*A(K-1,3)
    B(K)=B(K)-P(K-1)*B(K-1)
    A(K,3)=P(K)/A(K,2)
    B(K)=B(K)/A(K,2)
    Q=D(M-2)/D(M-1)
    A(M,1)=1+Q+A(M-2,3)
    A(M,2)=-Q-A(M,1)*A(M-1,3)
    B(M)=B(M-2)-A(M,1)*B(M-1)
    Z(M)=B(M)/A(M,2)
  MN=M-2
  DO 6 I=1,MN
    K=M-I
    Z(K)=B(K)-A(K,3)*Z(K+1)
    Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
  DO 7 K=1,MN

```



```

Q=1./ (6.*D(K))
C(1,K)=Z(K)*Q
C(2,K)=Z(K+1)*Q
C(3,K)=Y(K)/D(K)-Z(K)*P(K)
7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
  RETURN
  END

```


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13. ABSTRACT <p>Aircraft minimum flying speed, as determined by actual flight test, is published in aircraft handbooks for pilot guidance. The test flight results are used to determine and confirm take-off and landing speeds, field lengths, left-hand portion of the maneuvering envelopes (V-n diagram), etc. Determination of the absolute minimum flying speed of an aircraft on the other hand, has not been of prime importance in flight test.</p> <p>In the present analysis digital simulation allowed the systematic study of not only the minimum flying speed as defined by Federal Aviation Regulations but also the absolute minimum flying speed attainable in steady, unaccelerated flight. The study included such effects as deceleration rate, rate of change of elevator angle, aircraft weight and pitch moment of inertia.</p> <p>It was found for an assumed light-weight fighter aircraft that the absolute minimum flying speed was approximately 20 knots less than the FAR minimum flying speed. Moreover the FAR minimum flying speeds tended to be quite sensitive to rate of change of elevator angle.</p>			

14.

KEY WORDS

LINK A

LINK B

LINK C

ROLE

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ROLE

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ROLE

WT

Absolute Minimum Flying Speed

Stall Speed

FAR Minimum Flying Speed

Certified Minimum Flying Speed

Non-Linear Equations of Aircraft Motion

Digital Simulation of Aircraft Motion

Fourth-Order Runge-Kutte Algorithm

F-94A Stall Characteristics

Thesis
S185 Saul
c.1

134809

Evaluation of minimum
aircraft flying speed
by digital simulation.

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